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DRYING OF EAR CORN
Part II : THIN LAYER DRYING EQUATIONS

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Abstract

The objective of this paper was to investigate the drying behaviour of ear corn in thin layer. The first part was concerned with the determination of the effect of drying parameters on the drying process while this part is concerned with describing the thin layer drying mathematically

A simple exponential equation (Hukill, 1947) and its modified form (Page's equation, 1949) were used to describe thin layer drying of ear corn taking into accont two kinds of equilibrium moisture contents (dynamic equilibrium and final moisture content).

The results showed that page's equation (with thermodynamic equilibrium moisture content) was the best equation in describing the drying behaviour of ear corn. The resulting constants in the different equations were correlated with the experimental variables.

Introduction

Many of the research efforts devoted to grain drying have been concerned with the development of mathematical models for describing thin layer drying process. However, most of these models concerned with the drying of individual kernels. Mathematical models for describing thin layer drying of ear corn were tried through a limited number of studies.

Hassler (1959), Van Arsdel and Coply (1963) and Hall (1971) reported that the most of drying models were based on molecular diffusion which had been accepted as the basis of the drying theory where vapor pressure or moisture concentration was considered as the driving force causing moisture to move out of the solid.

Henderson and Perry (1955), Hall (1971) and Brooker *et, al.*, (1974) and many others have recognized that Fick's law of diffusion may be considered as the theoretical basis of the drying equations

$$F = -D\frac{dc}{dx} \qquad (1)$$

where,

F: rate of diffusion transport from a unit area.

D: liquid diffusion coefficient, m^2/min.

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c: concentration of diffusion substance.

x: space co-ordinate.

Fick's second law, which defines the rate change of concentration with time, may be drived from the previous equation as follow:

$$\frac{dc}{dt}=D\left[\frac{d^2c}{dx^2}+\left(\frac{j}{x}\right)\left(\frac{dc}{dx}\right)\right]\ldots\ldots(2)$$

where:

j: constant equal "o" for an infinite slab, equal "1" for an infinite cylinder and equal "2" for a sphere.

Crank (1964) found the analytical solutions of the previous equation for the average moisture content of various regularly shaped bodies. The solution for a sphere (j=2) could be achieved if the concentration is integrated over the volume to get the average moisture content instead of concentration:

$$\frac{M-M_e}{M_o-M_e}=\frac{6}{\pi^2}\sum_{n=1}^{n=\infty}\frac{1}{n^2}\exp\left[-\frac{n^2\pi^2D}{r_o^2}t\right]\ldots\ldots(3)$$

where:

t: the drying time.
r_o: the radius of drying particle, m.
n: number of molecules.

Based on an analogy to heat flow where Newton's law of cooling is applied to a body with resistance to heat flow concenterated at the surface, Hall (1971) showed Hukill's equation (1947) in the common form as follows:

$$\frac{M-M_e}{M_o-M_e}=A.e^{-Kt}\ldots\ldots(4)$$

Nellist and O'Callahan (1971) also found that the previous equation gave better fit to the data, but the resultant, K values were not related to any of the drying parameters, so they concluded that this equation is of little use for predicting drying within a computer simulation of deep bed drying.

On the other hand, Matouk (1976), Syarief *et. al.,* (1984), Pathak *et. al.,* (1991), Bala and Woods (1992), Muhidong *et. al.,* (1992) and Basunia and Abe (1998) found that Page's equation, (1949) which may be presented in the following form:

$$MR = \exp(-kt^u)\ldots\ldots(5)$$

was better than Hukill's equation in describing the thin layer drying behavior of maize grain, sunflower seed, rapeseed, malt, kenaf and rough rice.

The general objective of thes present paper was an attempt to provide rational basis for the artificial drying system, in which forced heated air at constant temperature and relative humidity may be used to remove excess moisture from ear corn.

While the specific objectives were:

1. To develop and select the proper mathematical model which may describe the loss of moisture during thin layer drying.
2. Correlating the constants of the developed equations to the experimental variables.

Theoretical approach

In this study, thin layer drying of ear corn under different air temperatures and relative humidities was examined. Two types of drying equations were tested. The analysis based on using two kindes of equilibrium moisture contents which are, the thermodynamically equilibrium moisture content (M_e), the final moisture content of the drying run as an equilibrium value(M_f), and the initial moisture cntent (M_o).

So the first type of drying equation was the simple exponential equation which may be written as follow:-

$$a-\frac{M-M_f}{M_O-M_f}=.e^{-K_s t} \quad\text{...(6)}$$

$$b-\frac{M-M_e}{M_O-M_e}=.e^{-K_s t} \quad\text{...(7)}$$

where:

M_f: final moisture content.

M_e: equilibrium moisture content.

K_s: the drying constant.

The values of the drying parameters K_s, M_e and M_f were determined, calculated and correlated to the drying variables as mentioned in the first part of this paper " Determination of drying parameters" (Matouk et. al.,2001).

The second type of drying equation may be written as follows:-

$$a-\frac{M-M_f}{M_O-M_f}=.e^{-K_p t^u} \quad\text{...(8)}$$

$$b-\frac{M-M_e}{M_O-M_e}=.e^{-K_p t^u} \quad\text{...(9)}$$

where:

K_p and u: the drying constants.

The previous equation may be re-written as:-

$\ln(-\ln(MR)) = \ln K_p + u \ln(t)$……………………………(10)

To determine the values of Kp and u, a straight line was fitted by the least square method to the values of ln(-ln(MR)) and the corresponding drying times. The slope of the fitted line represents the constant u while the intercept represent the value ($\ln K_p$), where MR, the moisture ratio.

The values of the drying constants K_p and u are listed in table (1).

The dependence of the drying constants K_p and u on the experimental variables (air temperature and air relative humidity) was studied using simple and multiple regression analysis.

Materials, Apparatus and Experimental Procedure

1. Material:

Ear corn used in this study was freshly harvested ear corn variety three way cross (T.W.C. 310). It was obtained from agricultural crops development center in Sakha (Kafr El Sheikh Governorate). It had nearly constant initial moisture content which ranged from (38 to 41% d.b.). The ear corn lot were cleaned and kept in refrigerated room at 5°C temperature.

2. Apparatus:

A detailed description of the dryer has been given in the first part "Determination of drying parameters" (Matouk *et. al.*,2001).

3. Measurements:

Detailed description of the measurements of the experimental variables such as moisture content, air velocity, air temperature, air relative humidity and initial and final mass of ear corn sample has been given by Matouk *et. al.*,(2001).

Experimental Procedures

1. Thin layer drying of ear corn:

In conducting the thin layer experiment, the air temperature was set at approximately 45, 50, 55 and 60 °C, and the air relative humidity at about 30%, 40%, 50% and 60%. Before an experimental run was started the whole of the apparatus was operated with a dummy sample for at least two hours. This period of time was essential for the conditioned air to stabilize and air flow rate to be adjusted. After the air temperature, air relative humidity and air flow rate had been stabilized, an ear corn sample (which consisted of three cops each of them divided into two halves) was distributed over the drying tray, located directly inside the drying bed. At the same time three samples were taken in tins from both ends of each half for moisture determination. Each tin was then covered with its lid and used later for the determination of the initial moisture content. As soon as this was ready, the dummy drying tray was removed from the drying bed and quickly replaced by the sample tray. The output from the weighing balance, which indicates the weight changes of the sample were recorded every five minutes for the first hour. Then every ten minutes for two hours. Then every twinty minutes for three hours. Then every fourty minutes for four hours. And finally every one hr. The experiments were kept running until the weight loss had almost ceased, which indicated that the moisture content of the ear corn had approached equilibrium with the drying air. At the completion of each drying test the final weight of ear corn was assessed and then the ear corn were used to determine the final moisture content as explained before.

Results and discussions

1. Calculation of drying constants (K_p and u):

Based on the second form of Page's equation
$(\ln(-\ln(MR)) = \ln(K_p) + u \ln(t)$

The values of the drying constants were obtained as mentioned before. The computed values of the drying constants were listed in table (1).

Table (1): Values of drying constants for the simple and modified equations:

Run no.	Ta	RH	Based on using M_e		Based on using M_f	
			Kp	u	Kp	u
1	45	32.3	0.0021789	0.95493	0.00198492	1.011033
2	45	41.4	0.00196390	0.989467	0.00170204	1.0412
3	45	50.07	0.0018083	1.01755	0.00158734	1.0629
4	45	59.65	0.00175795	1.0292	0.0014246	1.08815
5	50	30.1	0.00245183	0.99096	0.0021389	1.0342
6	50	40	0.00217075	0.9982	0.00192911	1.0558
7	50	49.85	0.00195723	1.0132	0.0017253	1.0725
8	50	60.55	0.00181652	1.0377	0.00153916	1.0972
9	55	29.93	0.00295298	0.97963	0.0023503	1.03973
10	55	40.15	0.00247194	0.98975	0.00210297	1.05415
11	55	49.25	0.00218518	1.002	0.00199784	1.06945
12	55	60.1	0.00196574	1.01975	0.0017578	1.08695
13	60	30.17	0.00307581	1.0158	0.00286484	1.03867
14	60	40.3	0.00271023	1.02053	0.00248211	1.05395
15	60	47.05	0.00234986	1.0371	0.00213603	1.0788
16	60	55.6	0.00207708	1.05285	0.00206608	1.0871

I- Effect of drying parameters on constant K_p:

Figures 1, 2, 3 and 4 show the relation between the drying constant (K_p) and both of air temperature (t_a) and air relative humidity (RH) at constant level of air relative humidity (30%) and air drying temperature (45°C)respectively. Same patterns were also found at other levels of air relative humidities and other levels of air temperatures.

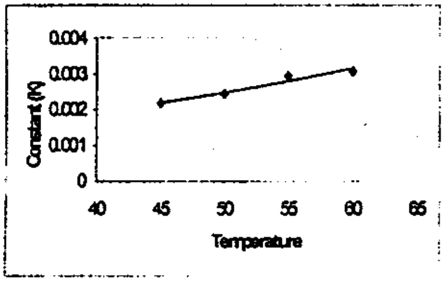

Figure (1):relation between (K_p) and (T_a) at constant level of air relative humidity of 30%

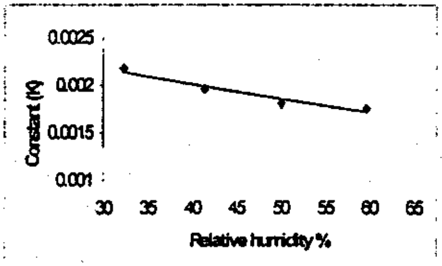

Figure (2):relation between (K_p) and (RH) at constant level of air temperature of 45°C

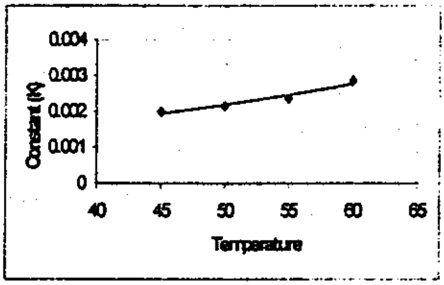

Figure (3):relation between (K_p) and (T_a) at constant level of air relative humidity of 30%

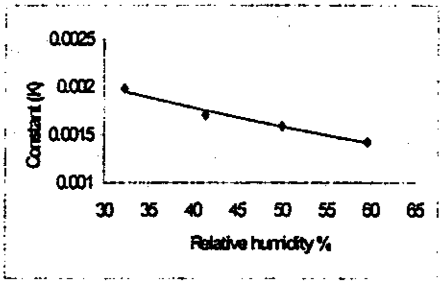

Figure (4):relation between (K_p) and (RH) at constant level of air temperature of 45°C

Simple regression analysis were then applied and the regression equations were as follow:

a- at a moisture ratio of $Mr=(M-M_e)/(M_o-M_e)$.

$K_p = A \exp(B\,(T_a))$..(11)

$K_p = C \exp(D\,(RH))$..(12)

b- at a moisture ratio of $Mr=(M-M_f)/(M_o-M_f)$.

$K_p = E \exp(F\,(T_a))$..(13)

$K_p = G \exp(H\,(RH))$..(14)

Tables (2) to (5) present the values of of the computed constants and coefficients of determination of equations (11) to (14) as found by regression analysis

Table (2): Values of constants (A, B) of equation (11) at constant levels of relative humidity.

RH	A	B	R^2
30%	0.00072664	0.024531	0.9511
40%	0.00072661	0.022030	0.9963
50%	0.00080616	0.017910	0.9938
60%	0.0010425	0.011414	0.9787

Table (3): Values of constants (C, D) of equation (12) at constant levels of air temperatures.

T_a	C	D	R^2
45 °C	0.0027689	- 0.00797894	0.9399
50 °C	0.0032544	- 0.00986665	0.9848
55 °C	0.0043409	- 0.01356137	0.9826
60 °C	0.0050062	- 0.01.584195	0.9901

Table (4): Values of constants (E, F) of equation (13) at constant levels of relative humidity.

RH	E	F	R^2
30%	0.00065668	0.023954	0.9480
40%	0.00056778	0.024305	0.9852
50%	0.00061927	0.020780	0.9828
60%	0.00044624	0.025184	0.9739

Table (5): Values of constants (G, H) of equation (14) at constant levels of air temperatures.

T_a	G	H	R^2
45 °C	0.0028358	- 0.011660196	0.9801
50 °C	0.0029873	- 0.011025851	0.9996
55 °C	0.0030994	- 0.009339183	0.9884
60 °C	0.0042544	- 0.013555821	0.9343

A multiple regression analysis was then applied to study the effect of air temperature and air relative humidity on the drying constant (K_p). The analysis showed that the nature of dependence may be expressed by the following equations:

a- considering M_e as the equilibrium moisture content:

$K = 0.001379 + 0.0000392\,T_a - 0.000027\,RH$ ....(15)

(S.E.= 0.000126 $R^2 = 0.914$)

b- considering M_f as the equilibrium moisture content:

$K = 0.000694 + 0.0000436\,T_a - 0.000022\,RH$ ....(16)

(S.E.= 0.0000924 $R^2 = 0.94706$)

II-Effect of drying parameters on the drying constant u:

similar analysis was proceeded to assess the relationship between constant (u) and both of drying air temperature and air relative humidity. The results showed that air temperatures almost had no effect on the drying constant u since the computed values of the drying constant (u) ranged from 0.95493 to 1.05285 (S.D.= 0.229219) using M_e as an equilibrium moisture value, while it ranged from 1.011033 to 1.0972 (S.D.= 0.023954) by using M_f as an equilibrium moisture content, so it could be considered as a constant value.

On the other hand, the values of the drying constant (u) showed dependence on the relative humidity of the drying air. The nature of dependence as computed by regression analysis may be described as follows:

a- at a moisture ratio of $Mr=(M-M_e)/(M_o-M_e)$.

$u = L + J\ RH + I\ RH^2$ ....(17)

Figure (5) reveals representative curve to show the relation between (u) and (RH) at temperature level of 45°C. Same patteren was also found at other levels of air relative humidity.

b- at a moisture ratio of $Mr=(M-M_f)/(M_o-M_f)$.

$u = V + S\ RH + R\ RH^2$ ....(18)

Figure (6) indicates representative curve to show the relation between (u) and (RH) at temperature level of 45°C. Same pattern was also found at other levels of air relative humidity.

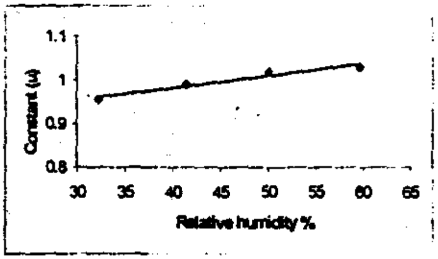

Figure (5): relation between (u) and (RH) at constant level of air temperature of 45°C

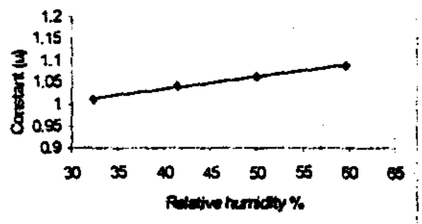

Figure (6): relation between (u) and (RH) at constant level of air temperature of 45°C

Tables (6) and (7) show the values of computed constants and coefficients of determination of equations (17) and (18) as found by regression analysis.

Table (6): Values of constants of equation (17) at constant levels of air relative humidity.

RH	I	J	L	R^2
30%	- 0.0000722759	0.0094060	0.72589	0.9975
40%	0.000038634	- 0.00196542	1.0151	1
50%	0.000016407	- 0.00014513	0.96924	1
60%	0.00004622	- 0.00244601	1.0468	0.9786

Table (7): Values of constants of equation (18) at constant levels of air relative humidity.

RH	R	S	V	R^2
30%	- 0.0000184135	0.004483	0.88587	0.9989
40%	0.0000038538	0.0016845	0.98057	0.9915
50%	0.000004982	0.0011266	1.0014	0.9987
60%	- 0.0000099492	0.0028767	0.95963	0.9541

The results of the previous analysis showed that, both constants of the modified exponential equation are significantly related to the experimental parameters. However, the air temperature has a higher effect on constant K_p in comparison with air relative humidity in both forms of moisture ratio.

On the other hand, the result showed that the effect of air temperatures on the drying constant u was not significant as the effect of air relative humidities. In other words one may conclude that the drying constant u was only affected by air relative humidity.

These results are in agreement with those reported by Sabbah (1968), Brooker (1974), Matouk (1976), and Hall (1980).

2. Thin Layer Drying Curves:

The decreasing in moisture content with the drying time as described by both forms of the simple exponential equation was affected by the drying parameters such as drying time (t), drying rate constant (K) and equilibrium or final moisture content (M_e or M_f). An accurate choice of these parameters is necessary for a better prediction of the change in the moisture content of ear corn with time.

In this respect, the two forms of the drying equations (simple equation and its modified form) were tested to find the best equation which can describe the drying data.

The results indicated that simple exponential equation (in which thermodynamically equilibrium moisture content was used) failed to describe the last portion of the drying curve in most cases. While simple exponetial equation (in which final mosture content was used as an equilibrium value) failed to describe the drying curve nearly in middle portion. Figures 7 and 8 show a representative typical drying behaviour.

On the other hand it can be seen (figures 7 and 8) that the modified exponential equation in which thermodynamically equilibrium moisture content was described the drying behaviour of the ear corn better than the modified exponential equation in which final moisture content was used.

In order to compare between the four drying equations, straight line was fitted by least square method to the values of the observed and calculated moisture contents. The values of standerd error of estemate and coefficient of determination were then computed. Figures 9 and 10 show the fitted straight line for the same representative runs which shown in figures 7 and 8.

Similar pattern was also noticed for all drying runs. Table 8 also shows the values of standerd error of estemate ond coefficient of determination, for all drying runs and all drying equations.

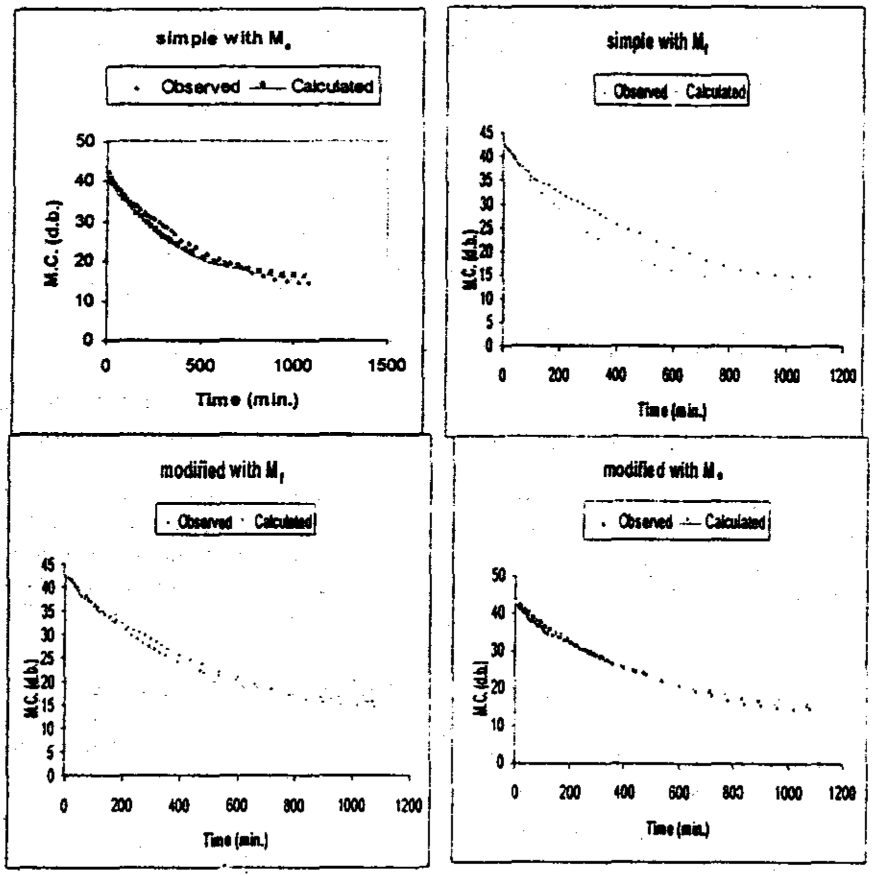

Figure (7): Observed and calculated values of M.C. obtained from simple and modified equations at 50°C and 60% RH

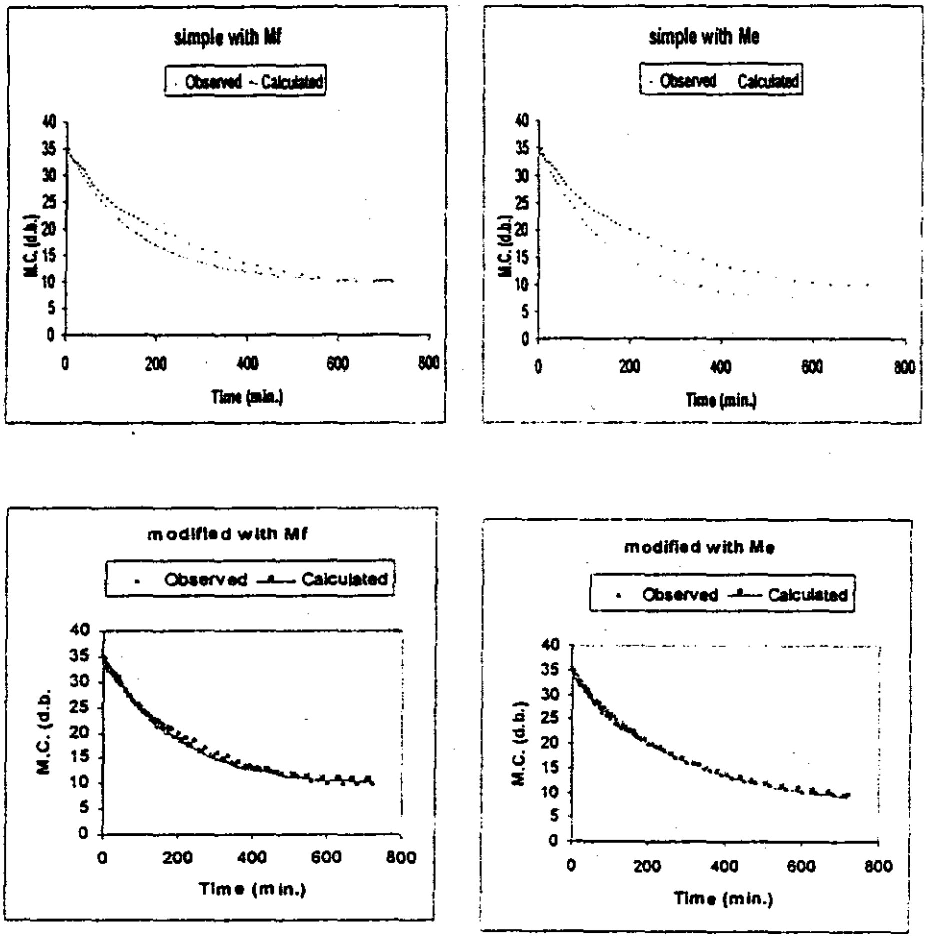

Figure (8): Observed and calculated values of M.C. obtained from simple and modified equations at 60°C and 30% RH.

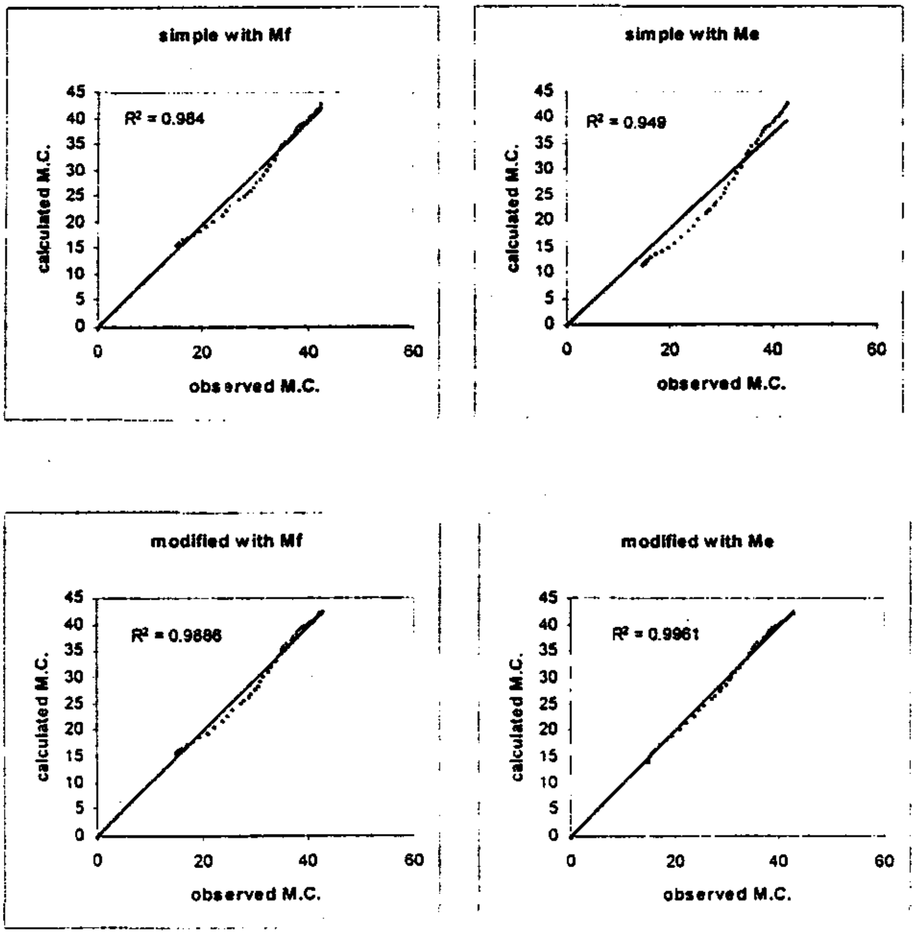

Figure (9): Observed and calculated values of M.C. obtained from simple and modified equations at 50°C and 60% RH.

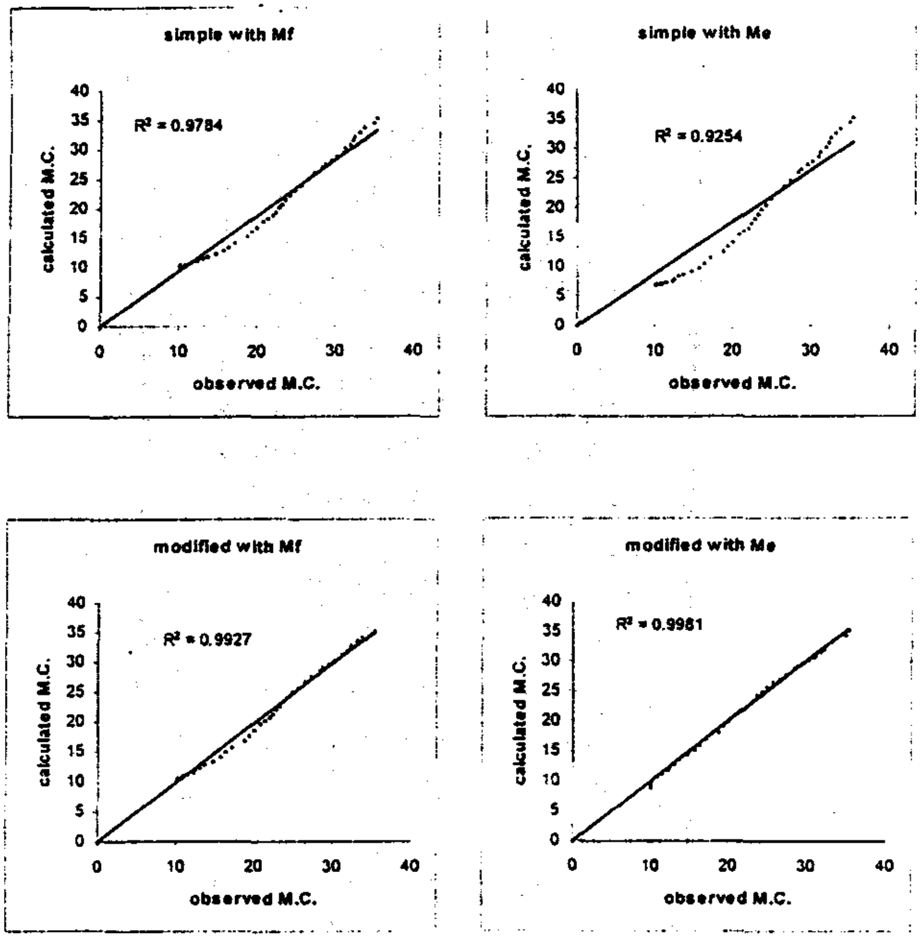

Figure (10): Observed and calculated values of M.C. obtained from simple and modified equations at 60°C and 30% RH.

Table (8): Values of coefficient of determination and the standard error of estimate for both forms of simple and modified equations.

Temp. (^oC)	RH (%)	Simple Using M_f		Simple Using Me		modified Using Mf		modified Using Me	
		S.E.	R^2	S.E.	R2	S.E.	R2	S.E.	R2
45	32.3	1.1127	0.9842	1.772	0.965	0.8027	0.9915	0.3029	0.9989
	41.4	1.9752	0.9692	2.4206	0.9586	1.521	0.9818	1.245	0.9875
	50.07	0.9095	0.9902	1.8102	0.9674	0.6166	0.9955	0.3345	0.9987
	59.65	1.465	0.9696	2.1466	0.9415	0.4785	0.9965	0.3509	0.9981
50	30.1	0.7089	0.9807	2.325	0.8595	0.512	0.9898	0.392	0.9943
	40	1.0183	0.9804	2.1887	0.9286	0.84648	0.9855	0.4579	0.9959
	49.85	1.1491	0.9797	2.1663	0.9426	0.964	0.9851	0.6129	0.994
	60.55	1.2717	0.984	2.517	0.949	1.0879	0.9886	0.6408	0.9961
55	29.93	1.702	0.9677	2.677	0.9316	0.8816	0.9912	0.475	0.9987
	40.15	1.2839	0.9842	1.9363	0.9682	1.1353	0.9866	0.9351	0.9908
	49.25	1.443	0.9653	2.3032	0.9265	0.7342	0.9908	0.3942	0.9974
	60.1	0.8442	0.9931	1.903	0.9604	0.7486	0.9948	0.4759	0.998
60	30.17	1.288	0.9784	2.6564	0.9254	0.757	0.9927	0.3817	0.998
	40.3	0.7462	0.9874	0.9328	0.9759	0.8212	0.9821	0.4603	0.9948
	47.05	0.6662	0.996	0.8111	0.9933	0.5985	0.9968	0.2352	0.9995
	55.6	0.3164	0.9978	1.0188	0.9732	0.6619	0.9909	0.3468	0.9974

A general comparison basied on the regression analysis between observed and calculated values of moisture content for both simple and modified exponential equations at different forms of moisture ratios to assess the most proper drying behavior of ear corn was made. The results showed that the modified equation (in which thermodynamically equilibrium moisture content was used) was the best equation in describing the drying behavior of ear corn, followed by the modified equation in which the final moisture content was used as equilibrium value, followed by the simple equation in which the final moisture content was used as equilibrium value and followed by the simple equation in which thermodynamically equilibrium moisture content was used as equilibrium value.

In other wards we may say that the modified exponential equation in the form of:

$(M-M_e)/(M_o-M_e) = \exp(-Kt^u)$

may be considered the most suitable equation in describing the drying behavior of ear corn during thin layer drying process.

conclusion

1. the modified exponential model in which thermodynamically equilibrium moisture content was used may be considered the most proper model for describing the drying behaviour of thin layer drying of ear corn.
2. the obtained mathematical relationships between the constant values of each model and the different drying parameters could be of great use in predicting and simulating the drying behaviour of a deep bed drying of ear corn.

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