J. Agric. Sci. Mansoura Univ., 28 (7): 5253 - 5268, 2003

A COMPARISON OF DIFFERENT MODELS OF THE LACTATION CURVE IN EGYPTIAN BUFFALOES

Aziz, M.A. [1], **A.E. Mahdy**[1], **O.M. El-Shafie**[2] **and N.A. Shalaby**[3]

[1] Department of Animal Production, Faculty of Agriculture, Alexandria University, Alexandria, Egypt.

[2] Animal Research Institute, Ministry of Agriculture, Cairo, Egypt.

[3] Department of Animal Production, Faculty of Agriculture, Al-Mansoura University, Al-Mansoura, Egypt.

ABSTRACT

Data on 933 records of weekly milk yield representing the first six lactations of 464 Egyptian buffaloes were utilized in this study. Buffaloes belong to the herd of the Animal Research Center of the Ministry of Agriculture located in Mahelet Mousa, Kafr El-sheikh Governorate. Data represented the period from 1989 to 1999. The objectives of the study were to describe lactation curve using the linear and the nonlinear forms of the incomplete gamma function and the inverse polynomial function and to compare the results obtained. Fitting the linear form of the incomplete gamma function gave logarithmic values (log a) for the initial milk yield ranged between 3.35 and 3.73 and the values increased with advancing parities from the first to the sixth. The rate of increase to peak production (b) ranged between 0.205 and 0.293 in the sixth and the first parities, respectively, however, the rate of decrease after peak ranged between 0.0192 and 0.0298 in the second and the first parities, respectively. All values calculated were significant ($P<0.01$). Percentages of R^2 ranged between 68% and 89%. Regarding the inverse polynomial function, the rate of increase to peak (A_0) ranged between 0.005 and 0.046 in the sixth and the second parities, respectively and the values were not significant. The slope of the lactation curve (A_1) ranged between 0.009 and 0.019 in the fifth and the third parities, respectively. The rates of decline after peak (A_2) were equal for all parities. The values of A_1 and A_2 were significant ($P<0.01$), except the slope (A_1) of the first parity. Percentages of R^2 ranged between 95% and 99%. On fitting lactation curve using the nonlinear form, the initial milk yield (a) ranged between 30.30 and 44.38 in the first and the sixth parities, respectively, and the values increased with advancing parity. The rate of increase to peak (b) ranged between 0.169 and 0.270, while the rate of decrease after peak ranged between 0.017 and 0.029. All parameters were significant ($P<0.01$) and percentages of R^2 ranged between 72% and 90%. Time to peak production (weeks), peak production (kg) and the fitted values of the total milk yield estimated by the three functions were compared with the corresponding values of the actual data. The nonlinear form of the incomplete gamma function gave better and closer results to the actual data than the linear form or the inverse polynomial function. Residual means squares (RMS) calculated by fitting the three functions were also compared. The results indicated that fitting the nonlinear form gave rise to the smallest RMS. It is concluded that the nonlinear form explained the shape of the lactation curve better than the linear form or the inverse polynomial function.

Keywords: Egyptian buffalo, lactation curve, incomplete gamma function, inverse polynomial function.

INTRODUCTION

Knowledge of the shape of lactation curve in dairy animals is important because the pattern of how an animal produces milk over time could determine its biological and economical efficiency for purposes of feeding, management and selection.

Since 1920's, there has been considerable interest in mathematical description and analysis of the lactation curve in dairy cattle. In this context, different models have been suggested to describe lactation curve (Wood, 1967; 1969; Batra,1986; Grossman and Koops, 1988 and Rook, *et al.* 1993). The wide variety of mathematical equations found in the literature results from a lengthy search for a robust form that performs well statistically over a wide variety of data sets, and that corrects minor biological defects perceived in previously applied forms. Unfortunately, no single study can completely answer the question of which mathematical equation will perform the best in a given application. Tozer and Huffaker (1999) stated that the search for a robust form has not resulted in any single equation emerging as the consistent best performer.

The linear form of the incomplete gamma function suggested by Wood (1967), has been the most commonly formula used for describing lactation curve in water buffalo (Samak, *et al.* 1988; Ibrahim, 1995 and Sadek *et al.* 1998.). On the other hand, few studies used the inverse polynomial function (Suhail *et al.* 1998). Furthermore, no reports were found in the literature on describing lactation curve in water buffalo using a nonlinear approach.

The objectives of this study were to describe lactation curve in Egyptian buffaloes using the linear and the nonlinear forms of the incomplete gamma and the inverse polynomial functions and to compare the results obtained.

MATERIALS AND METHODS

Data on 933 weekly milk yield records representing the first six lactations of 464 Egyptian buffaloes from 1989 to 1999 were utilized in this study.The records were collected from a herd belonging to the Animal Production Research Institute(APRI), the Ministry of Agriculture, Egypt. The herd is located in Mahelet Mousa, Kafr El-Sheikh Governorate. Numbers of records in the six parities were 166, 188, 175, 151, 140 and 113, respectively. Animals were fed according to the feeding system of APRI; Egyptian clover (Trifolium alexandrinum) during winter and spring. In summer and autumn, rice straw was available *ad lib*. All year round, they were offered concentrate mixture twice daily before milking according to their body weight and milk yield. Animals were naturally mated and milking was twice daily by hand. Weekly milk yield was the sum of the recorded daily milk production throughout the week. Data were edited such that buffaloes with 10 weeks of milk or less were excluded from the analysis. Maximum number of weeks in milk was 53.

Lactation curve was described by the following three methods:

1. The linear form of the Incomplete Gamma Function:

The function was suggested by Wood (1967) as follows:

$$Y_n = an^b e^{-cn}$$

Where
Y_n is the average milk yield in time period n,
a is a constant representing the level of initial yield of the buffalo cow,
b is a parameter representing the rate of increase to peak,
c is the rate of decline after peak
n is the time period, which is a week in this study
Taking the natural logarithm, the above formula becomes

$$\ln(y_n)=\ln(a)+b\ \ln(n)-cn$$

2. Inverse Polynomial function as described by Nelder (1966):

$$Y_n=n(A_0+A_1n+A_2n^2)^{-1}$$
$$n/Y_n= A_0+A_1n+A_2n^2$$

where
A_0 is the rate of increase to peak production,
A_1 is the slope of the lactation curve
A_2 is the rate of decline after the peak.
Both functions were fitted using PROC GLM of SAS (2000). Analysis was performed on each parity separately.

3. The nonlinear approach of the incomplete Gamma function, from (1). The Marquardt nonlinear algorithm of PROC NLIN (SAS, 2000) was used to estimate the equations for each parity. This method is particularly applicable when it is expected that the parameters to be estimated are highly correlated. The method is equivalent to performing a series of ridge regression, which correct for collinearity or near singularity problems that arise from the high correlation between the predictors of lactation curve (Bates and Watts, 1988). Fitting the nonlinear approach requires starting grid. The starting grid was specified such that all solutions fell within the outer limits of the search grid.

The three functions were chosen because they have been shown to be the most successful in fitting lactation data.

The resultant equations were compared based on statistical fit and how the fitted curves predict peak yield, weeks to peak, residual mean squares and the fitted values of the total milk yield over the length of lactation in comparison with the actual data. It was not possible to compare the results based on R^2, as the R^2 of nonlinear estimation cannot be used for hypothesis or significance testing (Pindyck and Rubinfeld, 1981).Time to peak was calculated by b/c for the incomplete gamma function and by $(b/c)^{1/2}$ for the inverse polynomial function. The yields associated with these times were obtained by substituting these times into the corresponding equation.

RESULTS AND DISCUSSION

Parameter estimates of the linear form of the incomplete gamma function for the six parities are presented in Table (1). Buffaloes in their first lactation had the lowest log(a). Log (a) increased with advancing parity, reaching its maximum in the sixth parity. The highest rate of increase to peak production (b) was in the first parity coincided with the highest rate of decline after peak (c). The lowest rate of increase to peak (b) was in the sixth parity, followed by that of the second parity, coincided with the lowest rate of decline (c). All parameters within each parity were significant (P<0.01).

Table (1): Lactation curve parameters estimated by the linear form of the incomplete Gamma function and their standard errors (se) and the coefficients of determination (R^2) for the six parities of Egyptian buffaloes. .

Parity Number	Log (a)±se	Exp (a)	B±se	c±se	R^2 (%)
1	3.35±0.08**	28.50	0.293±0.04**	0.0298±0.002**	83
2	3.48±0.03**	32.46	0.219±0.04**	0.0192±0.002**	68
3	3.54±0.03**	34.47	0.232±0.04**	0.0220±0.002**	72
4	3.63±0.03**	37.71	0.231±0.04**	0.0240±0.002**	81
5	3.65±0.03**	38.47	0.246±0.03**	0.0260±0.002**	89
6	3.73±0.03**	41.68	0.205±0.04**	0.0250±0.002**	85

** Significant at P<0.01

Log (a) is the logarithmic value of a and Exp (a) is the exponential value of log (a)
a is the initial yield, b is the rate of increase to peak production, c is the rate of decline after peak.

Lactation curve parameters estimated by fitting the inverse polynomial function for the six parities are shown in Table (2). The rate of increase to peak production (A_0) was not significant in all parities. The highest rate was in the second parity and the lowest was in the sixth parity. The slope of the lactation curve (A_1) of the third parity was the highest and the lowest was in the fifth parity. The estimates of the slope (A_1) were significant (P<0.01), except that of the first. The rates of decline after peak (A_2) of the six parities were equal and the values were all significant (P<0.01). The three parameters did not show certain trend across parities, as they oscillated throughout.

Table (2): Lactation curve parameters estimated by the inverse polynomial function and their standard errors (se) and the coefficient of determination (R^2) for the six parities of Egyptian buffaloes. .

Parity number	A_0±se	A_1±se	A_2±se	R^2 (%)
1	0.028±0.08	0.012±0.007	0.001±0.0002**	95
2	0.046±0.04	0.013±0.004**	0.001±0.0001**	96
3	0.012±0.03	0.019±0.002**	0.001±0.0001**	99
4	0.011±0.03	0.014±0.003**	0.001±0.0001**	98
5	0.042±0.03	0.009±0.002**	0.001±0.0001**	99
6	0.005±0.04	0.013±0.003**	0.001±0.0001**	98

** Significant at P<0.01

A_0 is the rate of increase to peak yield, A_1 is the slope of the lactation curve, A_2 is the rate of decline after peak.

Parameters of the nonlinear form of the incomplete gamma function are shown in Table (3). The initial yield (a) increased with advancing parity.

The lowest value (30.30) was in the first parity and the highest (44.38) was in the sixth parity. The rate of increase to peak (b) and the rate of decline after peak (c) increased with advancing parity starting from the second up to the fifth. Nevertheless, the highest rate of increase to peak (b) was in the first parity coincided with the highest rate of decline (c). The lowest rate of increase to peak and the lowest rate of decline after peak (c) were in the second parity. All parameters were significant (P<0.01). The parameters had similar trend to those estimated by the linear form (Table 1).

Table (3): **Lactation curve parameters estimated by the nonlinear form of the incomplete Gamma function and their approximate standard errors (se) and the coefficient of determination (R^2) for the six parities of Egyptian buffaloes.**

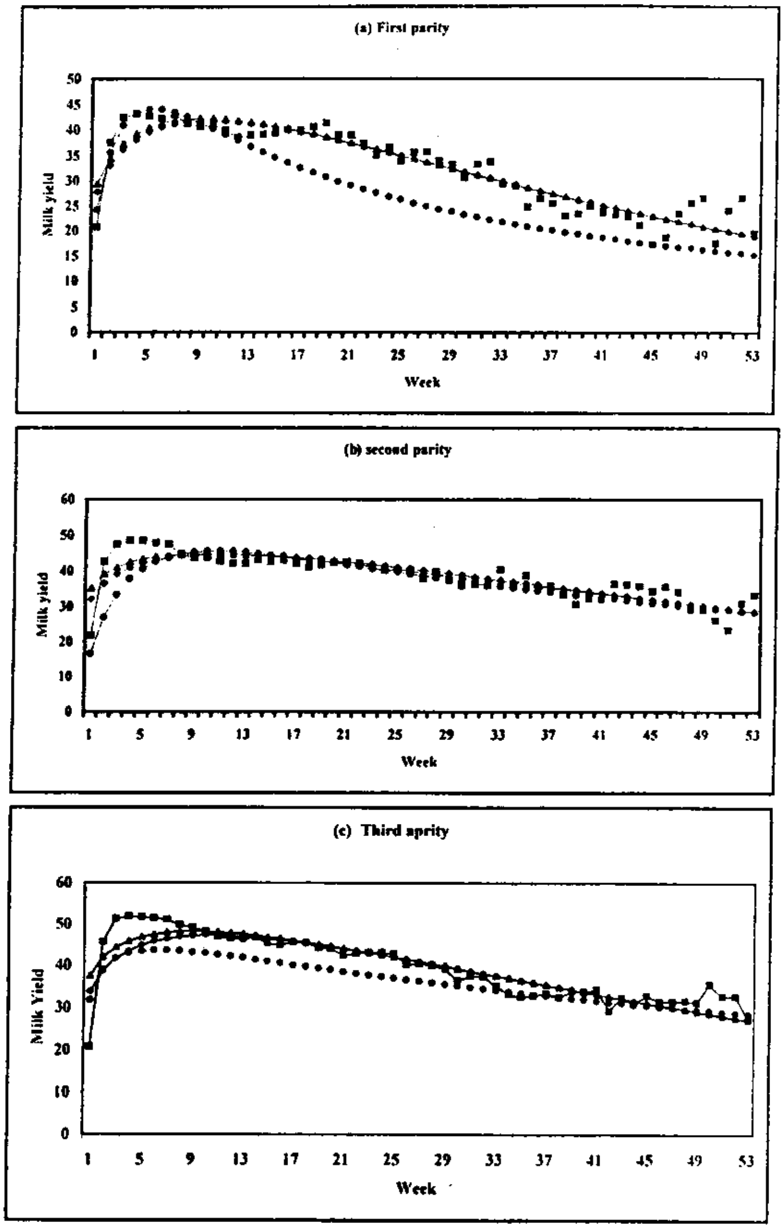
Parity Number	a±se	B±se	c±se	R^2 (%)
1	30.30±1.73**	0.270±0.034**	0.029±0.002**	87
2	35.91±1.98**	0.169±0.032**	0.017±0.002**	72
3	38.40±2.11**	0.189±0.032**	0.021±0.002**	79
4	41.04±2.04**	0.197±0.029**	0.023±0.002**	85
5	41.63±1.79**	0.215±0.026**	0.026±0.002**	90
6	44.38±2.22**	0.187±0.030**	0.026±0.002**	88

** Significant at P<0.01

a, b and c are the same as in Table (1).

Values of R^2 of the nonlinear form were slightly higher than the corresponding values of the linear form. When fitting the curve by the linear and the nonlinear forms of the incomplete gamma function, the lowest value of R^2 was observed in the second parity. The value of R^2 increased with increasing parity, reaching its maximum in the fifth parity. On the other hand, R^2 values of the inverse polynomial function were markedly higher than both values of the linear and the nonlinear forms of the incomplete gamma function, and ranged between 95 and 99% with no specific trend.

Fitted values obtained by the three functions for the six parities as well as the actual data are graphically presented in Figures (1, a-f). Averages of milk yield of the actual data fluctuated across weeks in all parities, especially throughout the first and the second parities. Lactation curves of the different parities described by the three functions had a long descending phase. This may be due to a long lactation period. In the first parity, the curve fitted by the inverse polynomial function had higher peak than those described by the linear and the nonlinear forms of the incomplete gamma function. The curve declined rapidly after peak compared to those described by the linear and the nonlinear forms that were close to each other. The inverse polynomial function described the initial rise in milk yield up to week 9 slightly better than did the linear and the nonlinear forms, then it underestimated milk yield up to the last week of lactation. The models of the linear and the nonlinear forms underestimated peak yield up to the ninth


(a) First parity
Milk yield
Week
(b) second parity
Milk yield
Week
(c) Third aprity
Milk Yield
Week

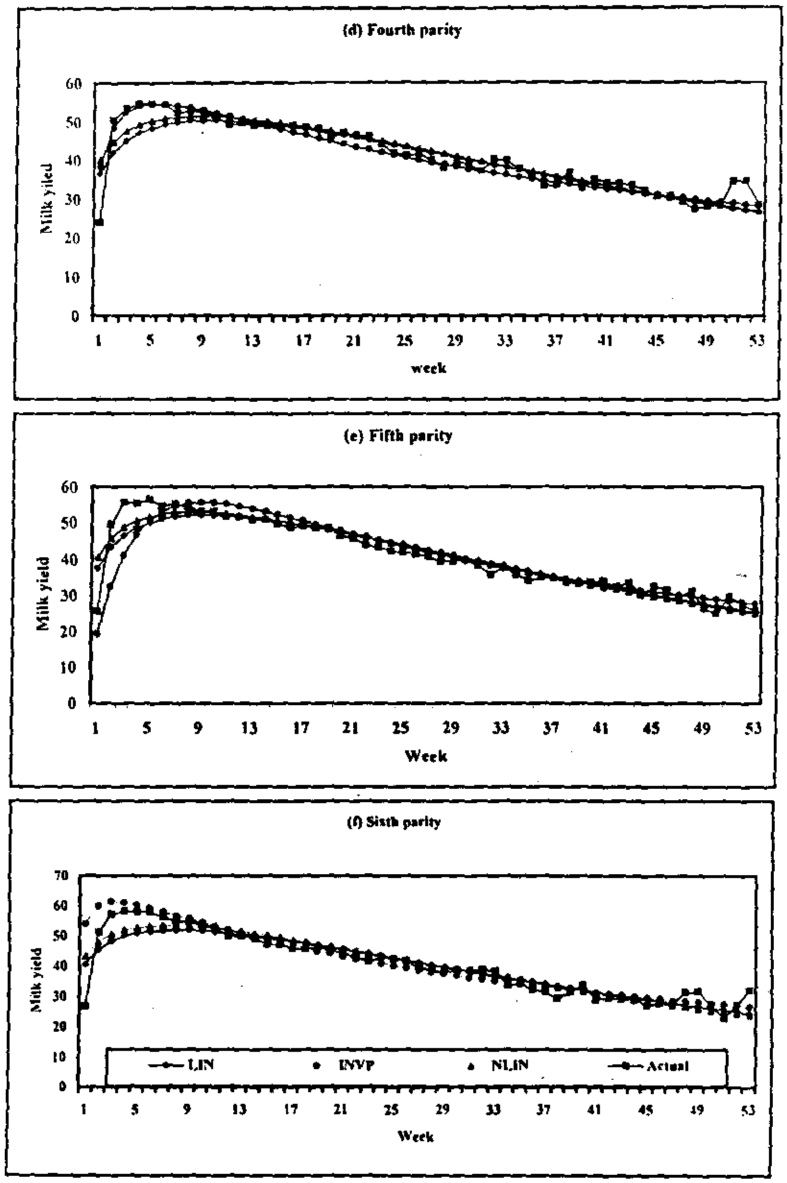


Figure (1): Lactation curve fitted by the linear form and the inverse polynomial and the nonlinear form of the incomplete gamma function and the actual data for the six parities of Egyptian buffaloes

Lin= Linear, INVP=Inverse Polynomial, NLIN=Nonlinear, Actual=actual data week, then they either underestimated or overestimated milk yield across the rest of the lactation period. All functions did not track the average weekly milk yield quite well (Figure 1a).

Lactation curves of the second parity resulted from fitting the three functions were more or less flatter than those of the other parities. The lowest values of the rate of increase to peak production (b) coincided with the lowest values of the rate of decline after peak (c) as estimated by the linear and the nonlinear forms confirmed this finding (Tables 1 and 3). In addition, the rate of increase to peak (A_0) in the second parity as estimated by the inverse polynomial function was higher than those of the other parities and the rate of decline after peak (A_2) was the lowest (Table 2). The curves had nearly similar graphical shape (Fig. 1b). Generally, the three functions underestimated the initial rise in milk yield from the second up to the ninth weeks. They overestimated milk yield from week 10 to week 21. Starting from week 27, the three functions were either underestimated or overestimated milk yield

In the third parity, the three functions underestimated actual milk yield up to the tenth week. Then, the linear and the nonlinear forms of the incomplete gamma function described the period from week 10 to week 32 better than did the inverse polynomial function. From week 33 onwards, the three functions underestimated or overestimated the actual yield (Fig. 1c).

The inverse polynomial function described the period of the first ten weeks of the fourth parity better than the two forms of the incomplete gamma function. The three functions provided better fit for the descending phase up to week 50 (Fig. 1d). The curves of the fifth and the sixth parities had similar trend to that observed in fourth parity with some exceptions. In the fifth parity, the three functions underestimated peak yield up to the ninth week. The inverse polynomial function overestimated milk yield from week 10 to week 18, while the two forms of the incomplete gamma function gave better fit for that period. From week 21 to week 36, the three functions slightly overestimated milk yield (Fig. 1e). In the sixth parity, the inverse polynomial function overestimated milk yield from the week one to week 13, while the two forms of the incomplete gamma function underestimated milk yield in the same period (Fig. 1f). In parities 4, 5 and 6, the three functions described milk yield from week 9 slightly better than they did in the first three parities. Actual milk yield of the first parity was poorly fitted by each of the three functions and the fit improved in the later parities.

Several workers had estimated lactation curve parameters for milk yield of different buffalo breeds. The most common method used was the linear form of the incomplete gamma function. Samak *et al.* (1988) described lactation curve of Egyptian buffaloes using the linear form of the incomplete gamma function. Log (a) estimates increased with advancing parity and ranged between 2.876 and 3.323 in the first and the fourth parities. Their estimates of the rate of increase to peak ranged between 0.421 and 0.537 and those of the rate of decline ranged between 0.036 and 0.050. Although log (a) values reported by the authors were lower than the corresponding values reported in this study, their values of b and c were higher. They reported values of R^2 ranged between 79 and 89, which are in close agreement with the findings of the present study. Ibrahim (1995) described lactation curve of Egyptian buffaloes by Wood's incomplete gamma function. He obtained values of the initial milk yield (a) ranged between 37.20 and 52.90. These values are higher than those reported in the present study, but the trend is similar, i.e. the initial yield (a) increased with advancing parity. The rate of increase to peak (b) in his study ranged between 0.31 and 0.37. The rate of decline after peak (c) ranged between 0.06 and 0.08. These values are higher than those reported in the present study. Sadek *et al.* (1998) used Wood's gamma function to estimate lactation curve parameters in two herds of Egyptian buffaloes. They found that initial milk yield (a) ranged between 35.5 and 52.3 kg in the first and the fourth parities, respectively. These values are higher than those reported in the present study. In their study, the initial rate (a) increased with advancing parity up to the fourth and then declined, which is in partial accordance with the trend observed in this study. In the present study, the initial rate (a) increased with parity up to the sixth. The rate of increase to peak (b) ranged between 0.27 to 0.36 in the fifth and the first parities, respectively. These values are higher than those reported in the present study. The rate of decline after peak (c) is also higher in their study and ranged between 0.05 and 0.07. Suhail *et al.* (1998) described lactation curve of Nili-Ravi buffaloes using the inverse polynomial function. They concluded that the function closely explained the descending phase of lactation, while it failed to explain the ascending phase efficiently. The estimated parameters (A., A_1 and A_2) in their study were 189.99, 0.0127 and 0.0092, which they are markedly higher than those estimated by the same function in the present study. The disagreement between the results of the present study and those reported in the cited literature could be due to different rates of milk secretion during the course of lactation resulting in a shape of lactation curve different in the present study from those found in the literature. According to Rao and Sundaresan (1979), the different rates of milk secretion during lactation are attributed to differences in the body condition of the animal at calving, feed availability and/or climatic conditions during lactations. It may also be due to breed differences.

It is difficult to decide which of the functions to recommend for use in practice. There has been no unique method that can be used in this regard. Based on R^2 criterion, Yadav *et al.* (1977) compared four models for fitting lactation curve (exponential, parabolic exponential, gamma and inverse polynomial) on the average weekly milk yield of Hariana cattle and its Friesian crosses. They reported that the inverse polynomial function provided the best fit, followed by the gamma function with R^2 values of 99 and 95, respectively. Batra (1986) also used the same criterion to compare lactation curve fitted by the inverse polynomial and the linear form of the incomplete gamma function and suggested that the former could be better than the later in describing lactation curve. However, he did not provide biological reason to support his suggestion, except that the function was previously utilized in growth and lactation research. In this regard, one may conclude that the inverse polynomial function is the method of choice. However, it is not correct statistically to compare the R^2 values of functions with different dependent variables.

Actual total milk yield and its fitted values estimated by the three functions for the six parities are presented in Table (4). Fitted values estimated by the linear and the nonlinear forms of the incomplete gamma function closer to the actual data than the fitted values estimated by the inverse polynomial function. Fitted values of the first and the third parities, estimated by the inverse polynomial function were greatly less than the corresponding values of the actual data. Both values represented 84 and 72% of the total milk yield of the actual data, respectively. Fitted values estimated by the nonlinear form approximately resembled the values of the actual data compared to those of the linear form. Furthermore, its trend was also similar to that observed in the actual data, where values increased with advancing parity up to the fourth. In the case of the linear form, the fitted values increased with advancing parity up to the fifth. However, fitted values of the inverse polynomial function increased with advancing parity starting from the third up to the sixth parities.

Table (4): Actual total milk yield and its fitted values (kg) estimated by the linear form, the inverse polynomial and the nonlinear form of the incomplete Gamma functions for the six parities of Egyptian buffaloes.

Parity Number	Function			Actual Milk Yield
	Linear	Inverse Polynomial	Nonlinear	
1	1694.00	1428.84	1710.66	1705.03
2	2023.22	1951.31	2024.45	2023.51
3	2066.61	1511.90	2093.59	2092.85
4	2140.02	2145.83	2176.21	2175.07
5	2164.49	2168.18	2168.97	2169.52
6	2126.28	2181.53	2135.84	2136.66

Time to peak production and peak yield estimated by the three functions and the actual data for the six parities are presented in Table (5). The present study indicated that time to peak production ranged between 8 and 12, between 3 and 11 and between 7 and 10 weeks for the linear form, the inverse polynomial and the nonlinear form of the incomplete gamma functions, respectively. The time to peak production was between 4 and 5 weeks in the actual data. The maximum number of weeks to peak production was in the second lactation and the minimum was in the sixth lactation. Starting from the second parity, time to peak calculated from the three functions decreased with increasing parity. In contrast, peak yield increased with advancing parity. The lowest peak production was in the first parity in all functions and the actual data as well. The highest peak was in the fifth parity for the linear form and in the sixth parity for the inverse polynomial, the nonlinear form and the actual data. Peak production estimated from fitting the nonlinear form was slightly closer to the actual peak than those obtained by fitting the linear form or the inverse polynomial functions. Percentages of peak production of the nonlinear form relative to that of the actual data ranged between 91 and 98%. The corresponding percentages of the linear form and the inverse polynomial functions ranged between 89 and 97 and between 87 and 105%, respectively. Fitting lactation curve by the linear and the nonlinear forms resulted in slight decrease in peak production relative to the actual data in the all parities, whereas the inverse polynomial function overestimated peak production in the first and the sixth parities by 2% and 5%, respectively. Samak *et al.* (1988) found that weeks to peak production in Egyptian buffaloes ranged between 10.7 and 13.6 weeks. These values are slightly higher than those reported in the present study. Ibrahim (1995) reported values of peak yield in the range of 46.5 to 64.0 kg. He also reported weeks to peak in the range of 5.2 to 6.5 weeks. Sadek *et al.* (1998) reported values ranged between 44 to 66 kg of peak yield. Weeks to peak yield ranged between 5.5 and 6.9. The results of the previous studies are in disagreement with the findings of the present study.

Table (5): **Time to peak (weeks) and peak yield (kg) estimated from fitting the linear form (LIN), the inverse polynomial (INVP), the nonlinear form (NLIN) of the incomplete gamma functions and the actual data for the six parities of Egyptian buffaloes..**

Parity Number	Time to peak (weeks)				Peak yield (kg)			
	LIN	INVP	NLN	Actual Data	LIN	INVP	NLN	Actual Data
1	10	5	9	4	41.7	44.1	42.2	43.2
2	12	11	10	5	44.7	45.5	44.7	48.5
3	11	7	9	4	47.2	45.0	48.3	51.9
4	10	5	9	5	50.4	54.6	51.5	54.8
5	9	9	8	5	52.1	55.6	53.0	56.5
6	8	3	7	4	52.0	61.3	53.4	58.2

Residual mean squares (RMS) estimated from fitting the three functions for the six parities are presented in Table (6).

Table (6): **Residual mean square (RMS) resulted from fitting the linear form (LIN), the inverse polynomial (INVP) and the nonlinear form (NLIN) of the incomplete gamma function to the actual data of the six parities of Egyptian buffaloes.**

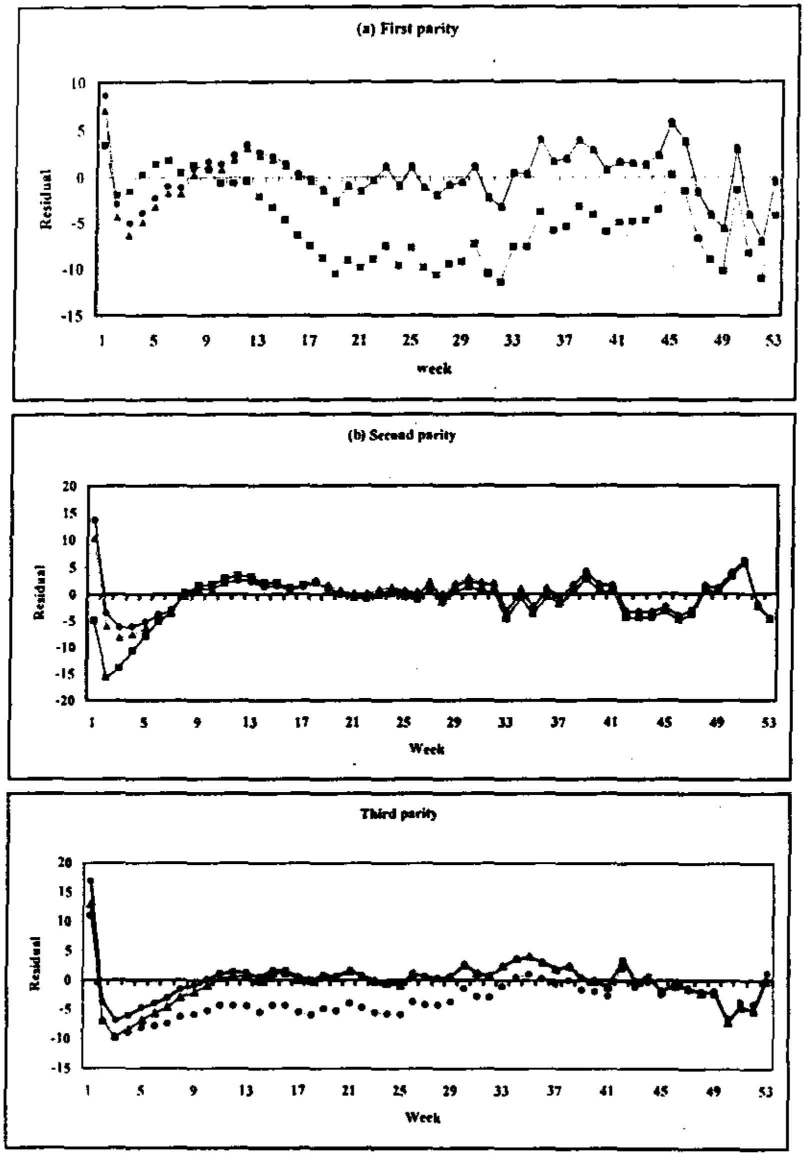
Parity Number	Residual mean square		
	LIN	INVP	NLIN
1	8.86	45.88	8.66
2	11.85	19.34	11.39
3	13.97	24.13	12.82
4	12.66	8.32	11.67
5	9.88	15.95	9.02
6	14.28	20.42	13.32

Residual means squares resulted from fitting the nonlinear form of the incomplete gamma function were slightly lower than those of the linear form, but they were markedly lower than those of the inverse polynomial function, except that of the fourth parity. The RMS values of the first, third and sixth parities resulted from fitting the inverse polynomial were markedly higher than the corresponding values of the linear and the nonlinear forms of the incomplete gamma function.

Residuals estimated by the three functions for the six parities are graphically presented in figures (2, a-f). For the fitted curve to give an adequate representation of the data, the residuals should show no evidence of trends but should be randomly scattered about the horizontal axis. This is clearly not so in all parities, but the problem is more severe in the first and the third parities. In the first parity, residuals resulted from fitting the linear and the nonlinear forms of the incomplete gamma function behave in a similar manner in comparison with those of the inverse polynomial function. Residuals of the first parity resulted from fitting the inverse polynomial function differed markedly in trend and in magnitude from those of the two forms of the incomplete gamma function.. An improvement in fitting the inverse polynomial function has occurred as parity progressed, where the residuals were getting closer to those resulted from fitting the two forms of the incomplete gamma function.

CONCLUSION

The results indicate that the two forms of the incomplete gamma function could be more appropriate than the inverse polynomial function in describing lactation curve and perform better with respect to the underlying process and the data. On the other hand, the nonlinear approach gave slightly better results than the linear one. It is preferred over the linear one as the parameters of lactation curve are essential nonlinear. The three functions predicts milk yield in earlier parities less accurately than that of the later parities, as indicated from the obvious improvement in fitting the curve in the later parities. One possible explanation for the poor fit in the earlier parities could be that the data were too variable. The nonlinear approach gave better estimates for peak production and total milk yield than did the linear approach and the inverse polynomial. Substantial reduction in RMS was observed when lactation curve was fitted by the nonlinear approach in comparison with the linear form and the inverse polynomial function. In dairy cattle, Cobby and Le Du (1978) stated that the nonlinear form of the incomplete gamma function gave better fit than the linear form.They based their conclusion on the substantial reduction in RMS achieved by fitting the nonlinear form of the incomplete gamma function. Rowlands *et al.* (1982) recommended using the nonlinear form of the incomplete gamma function for describing lactation curve in dairy cattle, where it gave rise to smaller residual mean square.


(a) First parity
Residual
week
(b) Second parity
Residual
Week
Third parity
Residual
Week

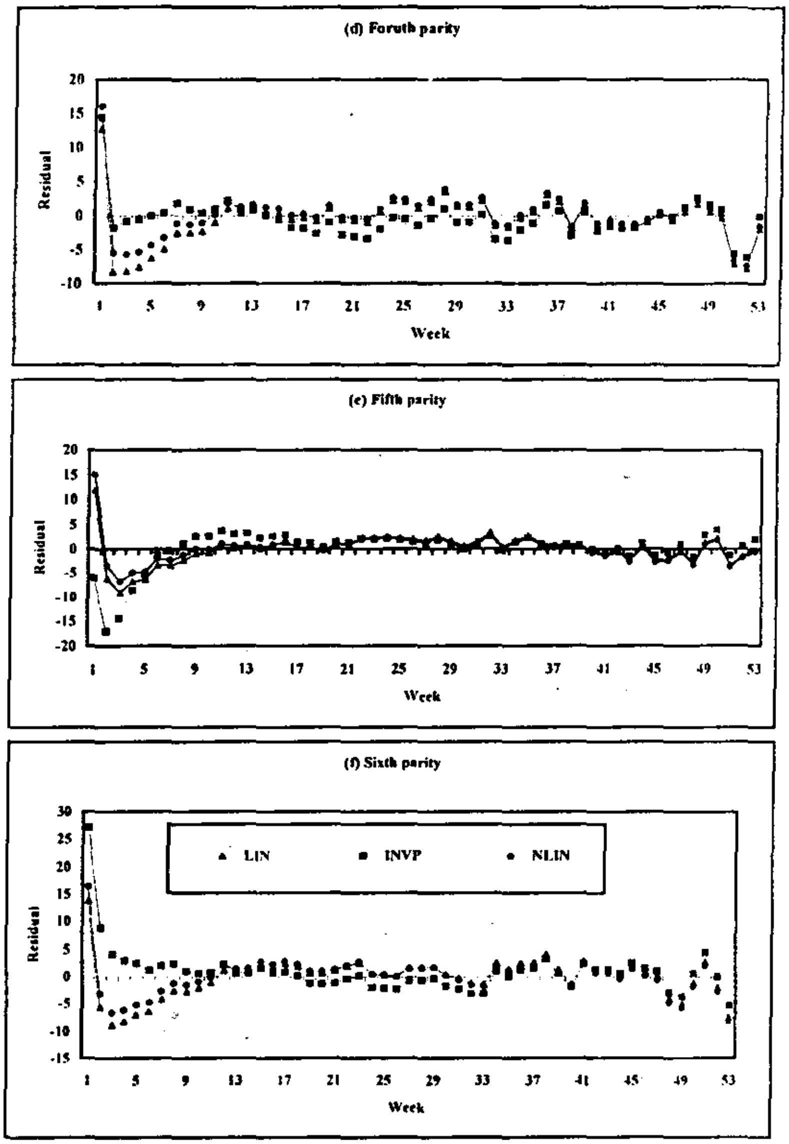


Figure (2): Residuals resulting from fitting the linear form. The inverse polynomial and nonlinear form of the incomplete gamma functions for the six parities in Egyptian buffaloes
Lin= Linear, INVP=Inverse polynomial, NLIN=Nonlinear

However, Tozer and Huffaker (1999) concluded that the incomplete gamma and the inverse polynomial functions were more robust in describing lactation curve in dairy cattle than other methods for describing lactation curve such as the Mitscherlich and the multiphasic functions. In water buffaloes, Tulloh and Holmes (1992) noted that the incomplete gamma function gave better fit than the inverse polynomial. According to the authors, the inverse polynomial underestimated milk yield during the initial stages of lactation and overestimated production during the middle stages, however, it gave a very close fit toward the end of lactation. In the present study, the inverse polynomial function overestimated the initial stage of lactation in the first and the sixth parities, and underestimated the same stage in the other parities. On the other hand, the two forms of the incomplete gamma function underestimated the same stage in all parities.

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مقارنة بين بعض الموديلات لوصف منحنى الحليب فى الجاموس المصرى

محمود أحمد عبد العزيز[١]، أحمد الطاهر مهدى[١]، عمر بهجت الشافعى[٢]، ناظم عبد الرحمن شلبى[٣]

[١]قسم الأنتاج الحيوانى، كلية الزراعة، جامعة الأسكندرية

[٢]معهد بحوث الأنتاج الحيوانى، مركز البحوث الزراعية، وزارة الزراعة

[٣]قسم الأنتاج الحيوانى، كلية الزراعة، جامعة المنصورة

استخدمت فى هذه الدراسة بيانات ٩٣٣ سجلا تمثل المواسم الستة الأولى لأنتاج اللبن الأسبوعى لعدد ٤٦٤ جاموسة حلابة، والتى تنتمى لقطيع الجاموس التابع لمعهد بحوث الأنتاج الحيوانى بمحطة محلة موسى. بمحافظة كفر الشيخ. جمعت بيانات هذه الدراسة خلال الفترة من ١٩٨٩ إلى ١٩٩٩. كان الغرض من الدراسة هو وصف منحنى الحليب للمواسم الستة باستخدام الدالة الخطية و الدالة غير الخطية لمعادلة Wood (Incomplete Gamma function) والدالة المعكوسة متعددة الحدود (Inverse Polynomial function)، و مقارنة النتائج المتحصل عليها. بالنسبة للدالة الخطية، أوضحت النتائج أن قيمة لوغاريتم مستوى الأنتاج الأولى (log a) تراوحت بين ٣,٣٥ و ٣,٧٣، حيث تزايدت القيمة من موسم الحليب الأول إلى موسم الحليب السادس. تراوح معدل أعلى مستوى أنتاج (b) بين ٠,٢٠٥ و ٠,٢٩٣ فى موسمى الحليب السادس و الأول على التوالى، أما معدل الأنخفاض فى مستوى الأنتاج (c) فقد تراوح بين ٠,٠١٩٢ و ٠,٠٢٩٨ فى موسمي الحليب الثانى و الأول على التوالى. كانت جميع القيم المتحصل عليها معنوية، وتراوحت نسبة معامل التحديد (R^2) بين ٦٨% و ٨٩%. بتطبيق الدالة المعكوسة متعددة الحدود، ترواح معدل أقصى أنتاج (A_0) بين ٠,٠٠٥ و ٠,٠٤٦ فى موسمى الحليب السادس و الثانى على التوالى، و كانت جميع القيم غير معنوية. تراوحت قيمة معامل أنحدار المنحنى (A_1) بين ٠,٠٠٩ فى موسم الحليب الخامس و ٠,٠١٩ فى موسم الحليب الثالث. تساوى معدل أنخفاض مستوى الحليب (A_2) فى كل المواسم (٠,٠٠١). كانت قيم A_1 و A_2 معنوية، ماعدا قيمة A_1 فى موسم الحليب الأول. تراوحت نسبة معامل التحديد (R^2) بين ٩٥% و ٩٩%. أما وصف المنحنى بإستخدام الدالة غير الخطية فأدى إلى تماثل شكل المنحنى بصورة تقريبية مع شكل المنحنى الذى وصف بإستخدام الدالة الخطية. تزايدت قيمة الأنتاج الأولى (a) بتقدم موسم الحليب، و تراوحت القيمة بين ٣٠,٣٠ و ٤٤,٣٨ فى موسمي الحليب الأول و السادس على التوالى. أما معدل الزيادة (b)، فتراوحت قيمته بين ٠,١٦٩ و ٠,٢٧٠ فى موسمى الحليب الثانى و الأول على التوالى. تراوحت قيمة معدل الأنخفاض (c) بين ٠,٠١٧ و ٠,٠٢٩ فى موسمى الحليب الثانى و الأول على التوالى أيضا. و كانت جميع القيم المحسوبة معنوية. تراوحت نسبة معامل التحديد (R^2) بين ٧٢% و ٩٠%. تم أيضا مقارنة الثلاث دالات عن طريق مقارنة عدد الأسابيع للوصول إلى أعلى مستوى أنتاجية محسوبا بإستخدام المعادلات الثلاث مع ذلك الفعلى، و كذلك مقارنة أعلى أنتاج محسوبا بالمعادلات الثلاث مع أعلى أنتاج فعلى. كذلك قورنت كمية اللبن الكلية الفعلية مع تلك المحسوبة بتطبيق المعايير المحسوبة بالثلاث دالات، و قد أعطت الدالة غير الخطية نتائج مقاربة للأنتاج الفعلى بدرجة أكبر من الدالة الخطية و الدالة المعكوسة متعددة الحدود. قورنت كذلك قيم متوسط مربع الأنحرافات للخطأ الناتج من الثلاث دالات، و قد أعطت الدالة غيرالخطية أقل متوسط لمربع الأنحرافات للخطأ مقارنة بالدالتين الأخريين. مما سبق استنتج أن الدالة غير الخطية قامت بوصف منحنى الحليب بدرجة أفضل من الدالتين الأخريين.