## MODELLING OF THIN LAYER DRYING OF GRAPES UNDER CONTROLLED CONDITIONS.

EL-SAHRIGI, A. F. <sup>1</sup>, A. M. MATOUK <sup>2</sup>, S. M. ABD EL-LATIEF<sup>2</sup>

#### AND A. SH. AWAD <sup>3</sup>

- 1. Fac. of Agric., Ain Shams Univ., Former Director of AENRI.
- 2. Fac. of Agric., Mansoura Univ.
- 3. Agricultural Engineering Research Institute, ARC, Egypt

(Manuscript received 31 October 2006)

#### **Abstract**

Two empirical models, exponential and Page were used to describe thin layer drying of grapes. The drying process was conducted under controlled drying technique at air temperature of 50, 60 and 70°C, air relative humidity of 15, 25 and 30% and air velocity of 2 m/s. Two forms of equilibrium moisture contents were considered which are thermodynamically in equilibrium, and final moisture content.

A set of empirical equations were also developed. The results showed that Page's model (with thermodynamically equilibrium moisture content) was the best model in describing the drying behavior of grape.

#### INTRODUCTION

Drying has been used world wide for centuries to preserve different kinds of food and other agricultural products. In Egypt as in most of Mediterranean countries dried grapes (raisin) represent a commercially important product.

It is difficult to obtain a universal drying model, by which the drying mechanism of heat and mass transfer of a material such as agricultural product can be described. Frequently, authors propose quite simple empirical models to simulate the drying curves of food that can provide adequate representation of experimental results although the parameters of these models lack the physical sense. The most simplified model is known as the simple exponential model (equation 1) based on an analogy to heat flow where Newton's law of cooling is applied to a body with resistance to heat flow concentrated at the surface. It has been used by many investigators (Hall (1971) and Matouk *et al.* (2002).

$$\frac{M - Me}{M_0 - M_0} = \exp^{-kt}$$
 .....(1)

#### Where:

M = average moisture content at time t, kg/kg

 $M_0$  = initial moisture content (d.b.), kg/kg

Me = equilibrium moisture content (d.b.), kg/kg

t = drying time, min

k = drying constant, min<sup>-1</sup>

On the other hand Pangavhane *et al.*, (2000), Matouk *et al.*, (2002) and Doymaz and Pala (2002) and Simal *et al.* (2005) reported that this model did not provide an accurate simulation of drying curves of many food products, under estimating the beginning of the drying curve and over estimating the later stages.

To overcome the short coming of the simple exponential model, the Page model, equation 2, is applied with an empirical modification to the time by introducing an exponent "n" as follows:

$$\frac{M - Me}{Mo - Me} = \exp^{-k_p t''} \dots (2)$$

Where:

 $K_p$ , n = drying constants.

This model has been used to accurately simulate the drying curves of ear corn (Matouk *et al.*, 2002), seedless grapes (Doymaz and Pala 2002), and carrot (Doymaz, 2004), among others.

The main objective of this study was to evaluate the two empirical models (simple experimental and Page) to simulate the drying curves of grape and propose a simple model to accurately simulate the drying kinetics of grape at different air temperatures (from 50 to 70°C) and relative humidities (from 15 to 35%).Also, to correlate the experimental variables within the scope of this study.

#### THEORETICAL CONSIDERATIONS

The value of moisture content of a solid after drying time, t, predicted by any drying equation depends mainly on the parameters (drying constant and equilibrium moisture content) of this equation. An accurate knowledge of these parameters in terms of the variables affecting them is necessary for better prediction to be made.

#### 2-1 Equilibrium moisture content

Two values of drying equilibrium moisture content for each run were considered in this study:

- a- The first value was the final moisture content of each run which was considered as an approximate value for dynamic equilibrium moisture content, taking into account that the drying run was turned off when the decrease in the sample weight was almost ceased. This value was denoted as M<sub>f</sub>.
- b- The second value determined according to Matouk et al., (2001) taking into consideration that, thermodynamically equilibrium occurs when the net exchange of moisture between the grape sample and air is zero, i.e., the rate of drying equals to zero. This value was denoted as M<sub>o</sub>.

#### 2-2 Drying models

As mentioned before two drying models were examined.

#### a- Simple exponential model

This model may be written as:

$$MR = \frac{M - M_c}{M_o - M_c} = \frac{M - M_f}{M_o - M_f} = \exp(-kt).......................(3)$$

Equation (3) was also simplified to straight-line model as:

$$In MR = -kt....(4)$$

#### b- Page model

Equation (2) which represents Page's model may be re-written in a linearized form as:

$$\ln (- \ln MR) = \ln (k_p) + n \ln (t)$$
 .....(5)

#### 2-3 Determination of drying constants

Drying rate constant (k) of the simple exponential model was obtained by applying linear regression analysis to the values of ln (MR) and elapsed drying time (t) (equation 4). The slope of the best fit straight line represents the value of the drying constant, k.

On the other hand the value of the drying constants of Page model,  $k_p$  and n, was obtained by applying linear regression analysis to the values ln(-ln(MR)) and the corresponding drying times (equation 5).

The slope of the fitted line represents the constant, n, while the intercept represents the value of the constant,  $k_{\rm p}$ .

#### MATERIALS AND EXPERIMENTAL METHODS

#### 3-1 Materials

Fresh, ripe hand harvested samples of seedless Thomposon (Banaty) were used for this study. It was obtained from Gemmeza Research Station in July (2003). The initial moisture content of the freshly harvested grape berries ranged from 77.3 % to 80.5 % (wet basis) and the sugar content was at Brix value of 23 as measured by the hand refractometer.

#### **Preparation of samples**

The grape samples were blanched by dipping in boiling solution containing 0.1 % NaOH for 10 seconds, and immediately cooled by immersing in running cold water. The blanched grapes were washed with tap water to be free of alkaline. Following this process, 35 g of sulfur powder was burned in a sulfuring cabinet for 8 hours to sulfur the blanched samples.

After the pre-treatment, the grapes samples were stored in plastic bags and kept inside a refrigerator at 5 °C until used.

#### 3-2 The laboratory dryer

The experimental dryer was designed to allow the control of air humidity and temperature and reduce turbulence of the air inside the drying chamber and ensure even distribution of the air around the sample tray. The drying chamber was also designed to provide easy handling of the sample tray and ensure minimal temperature gradient across the bed as shown in figure (1). Detailed description of the drying setup has been given by Matouk et al., (2001).

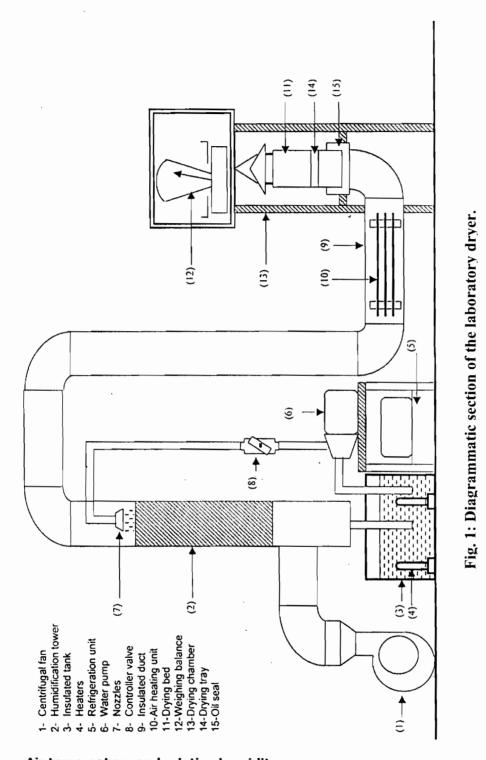
#### 3-3 Measuring Equipment

#### - Air velocity

The velocity of drying air through the trays was measured using a digital air velocity instrument connected to a velocity probe with measuring range from (0.1 to 10 m/s) and accuracy of ( $\pm 0.1 \text{ m/s}$ ).

#### - Initial and final weight

The weight of the samples at the beginning and end of the experiments was obtained by using a weighing balance accurate to 1gm.



### - Air temperature and relative humidity

A temperature and relative humidity meter was used for measuring both parameters during the experimental work. The measuring temperature range of the meter is from (-40 to 104.4 °C) with accuracy of ( $\pm$  0.2 °C). While the measuring range for relative humidity is from (10 to 95 %) with accuracy of ( $\pm$  1.5 %).

#### **Experimental Procedure.**

The experimental work was conducted at three levels of drying air temperature (50, 60 and 70 °C), three levels of air relative humidity (15, 25 and 35 %) and air velocity of 2 m/s. To make sure that the experimental unit accurately works before each experimental run it was operated with a dummy sample for at least two hours. After it was clear that air temperature, air flow rate and air relative humidity had been stabilized; the pretreated grapes were distributed uniformly at a single layer over the sample tray which was then placed directly inside the drying bed. At the same time three sub samples each of 5 g were taken from the pretreated grapes to determine the initial moisture content using the drying oven method recommended by (AOAC, 1995). The output from the weighing balance, indicated the weight changes of the samples which were recorded every 5 minutes along the first three hours, then every 10 minutes along the followed three hours, then every 20 minutes along the followed three hours, then every 40 minutes along the followed four hours and finally every 1 hour till the end of the run.

Each experiment was kept running until the weight loss had almost ceased, which means that the moisture content of the sample had approached equilibrium state with the drying air. At the end of each drying test the final weight of grapes was also assessed. In order to minimize the experimental errors of each run, it was replicated three times, and the average was considered.

#### **RESULTS AND DISCUSSION**

#### 4-1 Influence of drying air parameters on grape moisture content.

Figure (2) illustrates the change in grape moisture content as related to drying time at different levels of drying air temperature and air relative humidity. It can be seen that the rate of reduction in grape moisture content increased with the increase of drying air temperature, while it decreased with the increase of air relative humidity. Meanwhile no constant drying rate period was detected for all studied levels of drying air temperature and relative humidity while all the drying process occurred during the falling rate-drying period. This observation was in line with those noticed by Pangavhane, et al. (2000); Yaldiz et al. (2001) and Togrul and Pehlivan (2004).

# 4-2 Effect of drying parameters on the values of final and equilibrium moisture content

As mentioned before two values of dynamic equilibrium moisture content of each run were considered. The values pf  $M_f$  and  $M_e$  were estimated and tabulated in table (1).

As shown in table (1) both the final moisture content  $(M_f)$  and the equilibrium moisture content  $(M_e)$  decreased with the increase of drying air temperature while they increased with the increase of air relative humidity.

A regression analysis was proceeded to examine the nature of dependence between both equilibrium and final moisture content and drying parameters.

The regression analysis showed an opposite linear relationship between  $(M_f)$ ,  $(M_e)$  and the drying air temperature (t) for all levels of air relative humidity and velocity. While a direct linear relationship was obtained between the drying air relative humidity and both  $(M_f)$ ,  $(M_e)$  for all levels of drying air temperature. Figs 3 and 4 show the relation between the estimated values of  $M_f$  and  $M_e$  and drying parameters.

Meanwhile, to determine the interaction effect of the studied parameters on both  $(M_f)$  and  $(M_e)$  values, a multiple regression analysis was also employed. It was found that, both final and equilibrium moisture contents are strongly dependent on air temperature and relative humidity as follow:

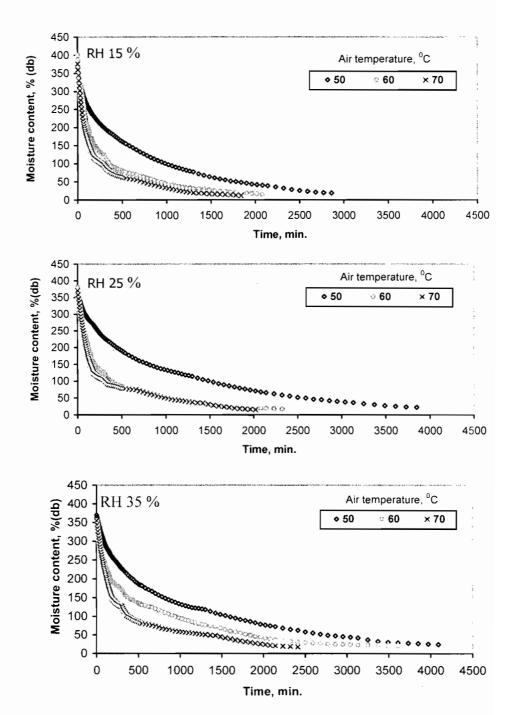


Fig. 2. Grape moisture content as related to drying time at different drying air temperatures, relative humidities and air velocity of 2 m/s.

N. B. The computed values of the drying constant (k) are listed in table (1).

Table 1. Calculated values of drying constant (k) of simple experimental equation, final moisture content and the thermodynamically equilibrium moisture content at different drying parameters

Run	Air	Air relative	Final	Thermodynami-cally	Values of k	Values of k	
No.	, temp.	humidity RH, %	moisture	equilibrium Me, %	based on	based on M <sub>e</sub> ,	
	t, °C		content		M <sub>r</sub> , min <sup>-1</sup>	min <sup>-1</sup>	
			Mf, % d.b.				
1	50	15	18.737	14.000	0.0015	0.0014	
2	50	25	21.999	17.889	0.0011	0.0010	
3	50	35	24.146	19.750	0.0011	0.0010	
4	60	15	15.618	12.020	0.0043	0.00397	
5	60	_25	17.315	14.656	0.0039	0.0036	
6	_60	35	20.847	18.826	0.0027	0.0025	
7	70	15	11.838	8.683	0.0055	0.0050	
8	70	25	14.994	13.121	0.0043	0.0041	
9	70	35	17.575	15.020	0.0035	0.00327	

Figs.5 and 6 show the relation between the drying constant (k) and both air temperature and relative humidity.

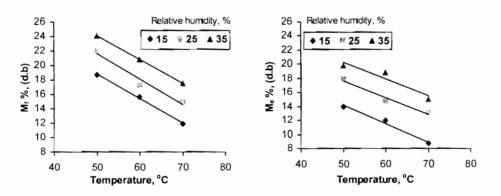
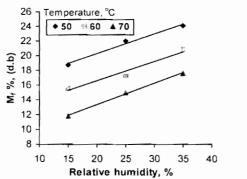


Fig. 3. Relation between " $M_{\rm f}$ ,  $M_{\rm e}$ " and drying air temperature "T" at different air relative humidities.



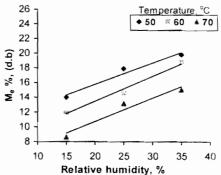


Fig. 4. Relation between  ${}^{``}M_{f}$ ,  $M_{e}$  and air relative humidity  ${}^{``}RH''$  at different air temperatures.

#### 4-3 Drying constants k, kp and n

#### a- Effect of drying parameters on the constant k of the simple model

Based on the logarithmic form of the simple exponential equation:

$$Ln MR = -kt$$

A multiple regression analysis was then applied to study the effect of the studied parameters (T and RH) on the drying constant (k) of the simple exponential equation. The analysis showed that the nature of dependence may be expressed by the following equation:

- considering M<sub>f</sub> as the equilibrium moisture content

$$k = -0.00483 + 0.00016 (T) - 0.000067 (RH)-----(8)$$
  
[SE = 0.000596;  $R^2 = 0.89418$ ]

- considering Me as the equilibrium moisture content

$$k = -0.0046 + 0.00015 (T) - 0.00006 (RH)-----(9)$$
  
 $SE = 0.000528 ; R^2 = 0.9029161$ 

The above mentioned analysis revealed that the constant (k) was highly dependent on both air temperature and relative humidity when using both final and equilibrium moisture content. These results are in agreement with Simal et al. (2005) and Amir (1999).

# b- Effect of drying parameters on drying constants $\mathbf{k}_p$ and $\mathbf{n}$ of Page's model.

Based on the second form of Page's equation:

$$(Ln (-Ln (MR))) = Ln (kp) + n Ln (t)$$

The value of the constant  $k_p$  and n were determined as mentioned in section (2-3). The values of the constants  $k_p$  and n were tabulated in table (2).

Table 2. Calculated values of the constant  $k_{\text{p}}$  and n of Page's model at different drying parameters.

Run No.	Based on M <sub>f</sub> ec	quilibrium value	Based on M <sub>e</sub> equilibrium value			
(temp., humidity)	k <sub>p</sub>	n	k <sub>p</sub>	n		
1 (50°C,15%)	0.008998	0.7512	0.00947	0.7368		
2 (50°C,25%)	0.006956	0.7621	0.00727	0.7406		
3 (50°C,35%)	0.005791	0.7874	0.00598	0.7755		
4 (60°C,15%)	0.011603	0.8540	0.01259	0.8239		
5 (60°C,25%)	0.009563	0.8647	0.01045	0.8406		
6 _(60°C,35%)	0.006683	0.8698	0.007241	0.8487		
7 (70°C,15%)	0.01376	0.8619	0.01499	0.8355		
8 (70°C,25%)	0.010108	0.8766	0.01136	0.8626		
9 (70°C,35%)	0.008553	0.8879	0.00914	0.8679		

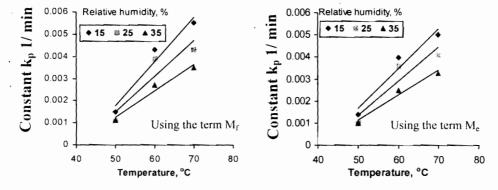


Fig. 5. Relation between " $k_p$ " and "T" at different air relative humidities using the terms ( $M_f,\ M_e$ ).

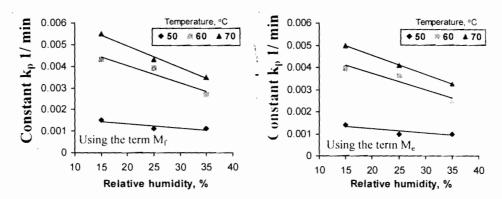


Fig. 6. Relation between " $k_p$ " and "RH" at different air temperatures and using the terms ( $M_f$ ,  $M_e$ ).

Inspection of the data of table (2) revealed that the constant  $k_p$  increased with the increase of air temperature while it decreased with the increase of air relative humidity as shown in figs 7 and 8.

Multiple regression analysis was employed to study the effect of air humidity on the drying constant  $(k_p)$ . The analysis showed that the nature of dependence may be expressed by the following equations:

- considering M<sub>f</sub> as the equilibrium moisture content

$$k_p = 0.003351 + 0.00019$$
 (T) - 0.00022 (RH)- -----(10)  
[SE = 0.000504 ;  $R^2 = 0.971164$ ]  
- considering  $M_e$  as the equilibrium moisture content

 $k_0 = 0.003183 + 0.000213 (T) - 0.00024 (RH) ----- (11)$ 

[SE = 
$$0.000647$$
;  $R^2 = 0.96172$ ]

On the other hand inspection of the data tabulated in table (2) showed that the values of the constant n ranged from 0.7368 to 0.8879. These results are in line with Abd El-Maksoud (2002) who concluded that the constant n depends on the type of the fruit and it was less than one for grapes.

Inspection of the data also revealed that the value of the constant n depends on both air temperature and air relative humidity.

A multiple regression analysis was also applied to study the effect of air temperature and air relative humidity on the drying constant n. The analysis showed that the nature of dependence may be expressed by the following equations:

- considering  $M_f$  as the equilibrium moisture content  $n = 0.476867 + 0.005428 \, (T) + 0.0013 \, (RH) ------(12)$  [SE = 0.024547 ,  $R^2 = 0.83795$ ] - considering  $M_e$  as the equilibrium moisture content  $n = 0.466369 + 0.005218 \, (T) + 0.001448 \, (RH) ------(13)$  [SE = 0.23113 ,  $R^2 = 0.845914$ ]

#### 4-5 Thin layers drying curves

The decrease in moisture with the drying time (drying behavior) as discussed by both models, simple exponential and Page, was found to be a function of the drying parameters such as drying time (t), drying constants (k,  $k_p$  and n) and dynamic equilibrium moisture content or final moisture content ( $M_e$  or  $M_f$ ). An accurate choice of these parameters is necessary for better prediction of the change in the grape moisture content to be made.

The results of this work indicated that no constant drying rate was detected for all studied levels of drying air temperature and relative humidity, while all the drying process occurred during the falling rate-drying period. The results also showed that both of the two models could describe the drying behavior of grape satisfactory. Figs 9 and 10 represent the drying behavior of some drying runs. Similar pattern was also noticed for the result of other runs.

It can also be seen (Figs 9 and 10) that Page's model in which thermodynamically equilibrium moisture content was used may be considered the most appropriate for describing the drying behavior of grapes.

In order to compare between the four drying equations (two models with two equilibrium values) straight line was fitted by least square method to the values of the observed and calculated moisture contents.

The values of standard error were estimated and coefficients of determination were then computed and tabulated in table (3). The results (table 3) also confirmed the above obtained results and showed that Page's model (in which thermodynamically equilibrium moisture content was used) was the appropriate equation in describing the drying behavior of grapes, followed by Page's model in which final moisture content was used as an equilibrium value, followed by the simple exponential model in which the final moisture content was used as an equilibrium value and followed by the simple exponential model in which thermodynamically equilibrium moisture content was used as an equilibrium value.

Table 3. \	/alues of ti	he coefficient	of dete	rmination	and t	the	standard	error of	estimate
	for both fo	rms of simple	and Pa	ge's mode	ls.				

Temp.,	RH, %	experimental expe		Sim experin	mental	Page model using		Page model using	
		SE	R <sup>2</sup>	SE	R <sup>2</sup>	SE	R <sup>2</sup>	SE	$\mathbb{R}^2$
	15	16.978	0.979	24.08	0.958	16.112	0.975	14.511	0.98
50	25	14.639	0.986	22.37	0.966	20.483	0.964	18.672	0.97
	35	20.229	0.968	22.48	0.961	10.58	0.99	8.494	0.994
	15_	18.962	0.97	22.45	0.945	14.918	0.977	15.881	0.972
60	25	14.793	0.979	22.351	0.953	11.619	0.985	13.169	0.98
	35	11.145	0.988	11.88	0.986	5.907	0.996	5.025	0.997
	15	17.393	0.972	19.8	0.964	11.127	0.986	10.326	0.988
70	25	19.943	0.963	22.046	0.955	13.195	0.981	12.865	0.982
	35	14.305	0.978	16.066	0.972	11.466	0.983	11.131	0.984

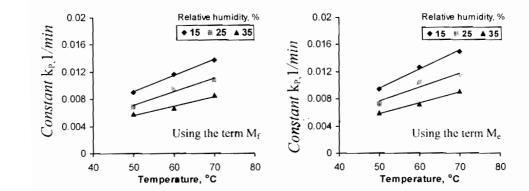


Fig. 7. Relation between " $k_P$ " and "T" at different air relative humidities using the term ( $M_f$ ,  $M_e$ ).

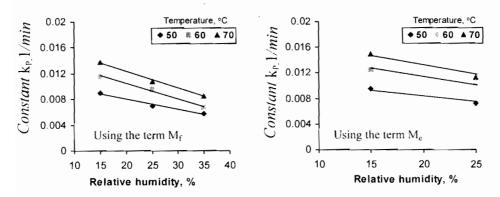


Fig. 8. Relation between " $k_P$ " and "RH" at different air temperatures using the term ( $M_f,\ M_e$ ).

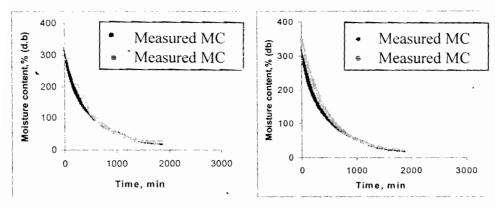


Fig. 9. Measured and predicted values of grape moisture content using the terms ( $M_f$ ,  $M_e$ ) at (T) 60° and (RH) 35 % for the simple equation.

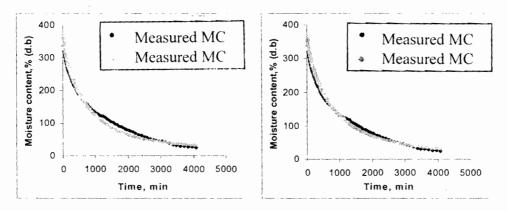


Fig. 10. Measured and predicted values of grape moisture content using the terms ( $M_f$ ,  $M_e$ ) at (T) 50° and (RH) 35 % for page equation.

#### CONCLUSION

The obtained results lead to the following conclusions:

- 1- Procedures to determine the drying parameters of grapes (drying constants and equilibrium moisture content) were developed.
- 2- Drying process of grapes occur during the falling rate drying period.
- 3- The Page's model in which thermodynamically equilibrium moisture content was used may be considered the most appropriate model for describing the thin layers drying behavior.
- 4- The obtained mathematical relationship between the values of constants of each model and the drying parameters may be of great importance in predicting and simulating the drying behavior in commercial dryers.

#### REFERENCES

- 1. *Abd El-Maksud, M. A. 2002.* "Utilization of green houses for fruit drying". Ph. D. Thesis, Ag. Mech. Dept., Fac. of Agric., Alex. Univ., Egypt.
- Amir, B.A.M., 1999. "Determination of drying rate of fruits as a function of the affecting factors under conditions suiting solar drying". M.Sc. Thesis, Ag. Eng. Dept, Fac. of Agric., Cairo Univ.
- AOAC. 1995. "Official methods of analysis of the association of official analytical chemists". 14<sup>th</sup> Ed. Published by the Association of Official Analytical Chemists, Arlington, Virginia, 22209 USA.
- 4. *Doymaz. I. and M. Pala. 2002.* "The effect of dipping pre-treatment on air drying rates of the seedless grapes". J. of food engineering, 52, 413-417.
- 5. *Doymaz. I., 2004.* "Convictive air drying characteristics of thin layer carrots". J. of food engineering, 61:359.364.
- 6. *Hall, C.W., 1971.* "Drying farm crops". The AVI. Publishing Company, Inc. Westerport. U.S.A.
- 7. *Matouk A. M., S. M. Abd El-Latif, Y. M. El-Hadidi, and A. Tharwat, 2001.* "Drying of ear corn part I: Determination of drying parameters". Misr J. of Agric. Eng. Vol. 18 (3): 805-820.
- 8. *Matouk A. M., S. M. Abd El-Latif, Y. M. El-Hadidi, and A. Tharwat. 2002.* "Drying of ear corn part II: thin layer drying equation". The 10<sup>th</sup> Annual Conference of the Misr Society of Agric. Eng., 16-17 October.: 139-154.
- 9. *Pangavhane, D. R., R. L. Sawhnew and P. N. Saravadia. 2000.* "Drying kinetic studies on single layer Thompson seedless grapes controlled heated air conditions". J. of food processing and preservation, 24, 335-352.
- 10. Simal, S., A. Femenia; C. M. Garau and C. Rossello. 2005. "Use of exponential, Page's and diffusion models to simulate the drying kinetics of Kiwi fruit". J. of food engineering, 66: 323-328.
- 11. *Toğrul, I. T. and D. Pehlivan. 2004.* "Modelling of thin layer drying kinetics of some fruit under open-air sun drying process". J. of food engineering, 65: 413-425.
- Yaldiz, O., C. Ertekin and H.I. Uzun. 2001. "Mathematical modelling of thin layer solar drying of sultana grapes". Energy an International J., 26: 457-465.

## نمذجة تجفيف العنب في طبقات رقيقه بالتحكم في ظروف التجفيف

# أحمد فريد السهريجي , أحمد محمود معتوق , صلاح مصطفى عبد اللطيف , علاء الدين شوقى عوض "

- ١. الهندسة الزراعية، كلية الزراعة جامعة عين شمس
- ٢. الهندسة الزراعية، كلية الزراعة جامعة المنصورة
  - ٣. معهد بحوث الهندسة الزراعية، الدقى جيزه

أجريت هذه الدراسة لوصف عملية التجفيف في طبقات رقيقة للعنب البناتي صنف (طومسون) وذلك بإجراء مجموعة من التجارب المعملية لدراسة تأثير مستويات مختلفة من درجة حرارة هواء التجفيف (٥٠, ٠٠, ٥٠°م), الرطوبة النسبية للهواء (١٥, ٥٠, ٣٥%) على عملية تجفيف العنب في طبقات رقيقة. وذلك بإستخدام مجفف معملي يمكنه التحكم في تلك المتغيرات.

تم إختبار نموذجين رياضيين لوصف منحنى التجفيف والتنبؤ بالتغير في المحتوى الرطوبي للعنب أثناء عملية التجفيف شملت (المعادلة البسيطه, معادلة Page).

## ولقد أظهرت النتائج المتحصل عليها ما يلي:

١ لم تظهر فترة معدل التجفيف الثابت لجميع منحنيات التجفيف وذلك عند جميع مستويات درجة الحرارة, الرطوبة النسبية, بينما تمت عملية التجفيف بالكامل خـــلال فتــرة معــدل التجفيف المتناقصة.

 $M_{\rm f}$  النهائي المتوازن  $M_{\rm e}$ ) والمحتوى الرطوبي النهائي  $M_{\rm f}$ ) النهائي النهائي النهائي  $M_{\rm f}$ ) القيم بزيادة درجة حرارة الهواء بينما زادت تلك القيم بزيادة الرطوبة النسبية للهواء.

 $^{-}$  أمكن لجميع النماذج المستخدمة وصف سلوك منحنى التجفيف والنتبؤ بالمحتوى الرطوبي بصوره جيده, بينما كانت معادلة (Page) الأفضل في وصف منحنى التجفيف والتنبؤ للمحتوى الرطوبي وذلك عند إستخدام المحتوى الرطوبي المتوازن ( $^{-}$ M<sub>e</sub>) بالمقارنة بالمحتوى الرطوبي النهائي ( $^{-}$ M<sub>f</sub>).