# Simulation of Soil Water Movement in Sandy Soil under a Prairie Field with Hydrus \_2D Model

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#### **ABSTRACT**

One of the main characteristics of trickle irrigation system is that water leaving an emitter enters the soil and moves both laterally and vertically. There has been much speculation on the shape and moisture distribution within the wetted soil volume. This knowledge is important in the design operation and management of a trickle irrigation. system. A simulation study of soil water distribution under a prairie field in Tripoli Libya, by the use of the two dimensional model (Hydrus 2D model) was carried out. Sandy soil was impated using surface point source with application rates of 1.5, 2, 2.7, 3, 3.5, 4.5, 4.8 and 6 Un. The surface wetted radius, vertical advance of wetting front and the distribution of moisture content in the soil profile were determined. Three statistical criteria were used to compare the quality of simulation results, namely, mean bias error (MBE), root mean square error (RMSE) and Their's inequality coefficient (U). Simulation positions of the wetting front estimated by the model were in agreement with the observed measurements of the wetting front for the lateral movement. In the downward direction, the simulated wetting front advanced much slower than the observed, especially at later stages of infiltration. Considering the difficulties in estimating the dynamic water conditions in the field, there was generally a good agreement (especially in the lateral direction) between the measured and simulated values. In the deeper downward direction, the simulated moisture content distributions were less than the measured. On the other hand, the Hydrus 2D model described the water content distribution quite well at relatively high levels of moisture contents; however, it did not do as well at lower moisture content. The discrepancies between the simulated and measured values may be due to the variation in the size of the surface point source of water during infiltration and to the natural variation of soil properties. The results support the use of Hydrus 2D as a tool for investigating and designing point source trickle imigation system.

Keywords: Trickle irrigation, wetting front, soil moisture distribution, Hydrus \_2D model

#### INTRODUCTION

The relation between population growth and increased demand for food and available water resources gives warning of persistent disorder. The signs of water shortage are quite clear not only in arid and semi arid areas but also in parts of areas known as rich in water resources. Water allocation is a topical issue due to increasing competition between urban, rural, industrial and environmental users. Irrigation practices are therefore coming under closer scrutiny and driving a requirement for more efficient water use in agriculture. One of the methods that can provide high water

use efficiency is trickle irrigation, if designed and installed correctly i.e., emitter spacing, flow rate and depth of installation (Phene, 1995).

The development of mathematical models for soil- water system has become the objective of extensive research during the last decades, mainly from soil physicists, agricultural and hydraulics engineers (Elmaloglou et al., 2003). Numerical simulation is an efficient approach to investigating optimal trickle management practices (e.g., Bresler et al., 1971; Meshkat et al., 1999; Schmitz et al., 2002; Claire et al., 2003; Cote et al., 2003). However, there have been very few, if any, studies showing that numerical simulations of trickle irrigation agree with field data, thus bringing into question the value of conclusions drawn from numerical simulations.

The objective of this work was to compare HYDRUS\_2D (Šimunek et al., 1999) simulations of water infiltration with field data. HYDRUS\_2D is a well-known Windows-based computer software package for simulating water, heat, and/or sofute movement in two-dimensional, variably saturated porous media. In this study the HYDRUS\_2D model is used to simulate water movement into sandy soil under a Prairie Field conditions.

#### MATERIALS AND METHODS

Field experiments were conducted on bare sandy soil located some 20 km south of Tripoli-Libya, along the way of International Tripoli Airport. The sandy soil was classified as loose, very friable, and highly drained, with very fine roots. The main physical characteristics of the soil are given in the Table 1.

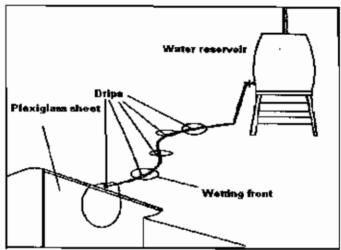
Table 1: Summary of soil physical properties used in the study

Şoil	Sand %	Silit %	Clay %	Bulk density, Mg/m <sup>3</sup>	(θν) <sub>1</sub> - (θν) <sub>2</sub>	Ks, m/d
Sandy	98	1	1	1.57	0.04 - 0.41	20.1

(8v), initial moisture content, (8v), saturated moisture content, Ks saturated hydraulic conductivity

Prior to conducting seach experiment, a Plexiglass sheet was installed alongside the wall of trenches dug to identify and to follow the advance of wetted front during water application. Efforts were required to level the soil surface where emitters were located. The trickle irrigation system used in this investigation was designed to provide uniform water application from a point source at various rates. Essentially, the apparatus consisted of a modified Marriott tube (Yitayew and Waston, 1986) was used as a reservoir joined to emitters by means of hose 0.75° ID (Figure 1). The

Emitters were calibrated for the application of the required volume at desired rate. The outlets were located on the soil surface where each dripper irrigated a distinct area without any interface with others; meanwhile the last dripper was located 10cm away of the edge of the trench. Various



application rates were chosen to be between 1.5 and 6 l/h. During each experiment, the advance of the wetted front on the soil surface and perpendicular face of trench was recorded periodically. After termination of the irrigation, a vertical section was made through the point where the source was located to determine the wetting-front position. This procedure was adopted in case of 2.0, 4.5, 4.8 and 6.0 l/h application rate. Due to the highly symmetrical shape of the wetting pattern, only one half of the wetted profile was sampled for water determination.

Figure 1: Field experimental setup

#### **Numerical Modeling**

Water movement in two direction models is complicated compared with one direction movement behavior. Numerical models are useful solutions to predict water and solute movement behavior in active root zone of the crop. Simunek et al., 1999 developed a general purpose two dimensional numerical model known as Hydrus\_2D. The Hydrus\_2D program numerically solves the Richards' equation (Eq. 1) for saturated and unsaturated water flow. The program is based on finite- element scheme and incorporates a graphical user interface for data entry (soil properties, atmospheric conditions, emitter discharge, and boundary conditions). The model will be used to compare the agreement between the measurements

and calculated values of water movement under point source trickle irrigation.

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x_i} \left( K^r(\theta) K_{ij}^A \frac{\partial h(\theta)}{\partial x_i} + K^r(\theta) K_{ii}^A \right) \pm S \tag{1}$$

Where  $\theta$  = the volumetric water content [L³/L³], h = the pressure head [L], S = a sink term [T¹]  $x_i$  (i=1, 2) = spatial coordinates [L],t = time [T],  $K'(\theta)$  = the relative hydraulic conductivity a function of  $\theta$ ,  $K_{ij}^A$  = the hydraulic conductivity tensor at saturation [LT¹].

In this study, h may be eliminated to have a "9-based" equation or  $\theta$  may be eliminated to have an "h-based" equation.

$$\theta = \theta_r + \frac{\theta_r - \theta_r}{\left[1 + \left| \frac{1}{2} h \right|^2\right]^{\alpha}}$$
 (2)

$$K(h) = \begin{cases} K_8K_n(h) & h < 0 \\ K_8 & h \ge 0 \end{cases}$$
 (3)

$$K(h) = K_{s}S_{s}^{1}[1 - (1 - S_{s}^{1/m})^{m}]^{2}$$
 (4)

$$m=1-(1/n)$$
  $n>1$   $Se=(\theta-\theta_r)/(\theta_s-\theta_r)$  (5)

I= the pore connectivity parameter  $[-], \theta_s =$  the saturated water content  $[L^3L^3]$   $\theta_r$  =the residual water content  $[L^3L^3]$ , may be defined as the water content at which both  $d\Theta/dh$  and K goes to zero when it becomes large.  $\alpha=$  an empirical factor  $[L^3]$ , whose inverse is referred to as the air entry value or bubbling pressure. n and m= the empirical factors affecting the shape of retention curve [dimensionless].  $S_a=$  the effective degree of saturation, also called the reduced water content  $(0 \le S_a \le 1)$ .

Estimating of Van Genuchten parameters from the experimental data requires; sufficient data points of  $\theta$  (h) pair (at least 5 to 8) and data points of k ( $\theta$ ) or D ( $\theta$ ) pair and a program for performing nonlinear regression.

Recent versions of many computer spreadsheets provide relatively simple and effective mechanisms for performing nonlinear regression. However, retention curve, RETC, (Van Genuchten et al., 1991) for describing soil water hydraulic properties can be used to evaluate the parameters used in Hydrus\_2D model (Šimůnek et al., 1999).

#### Problem Definition

Simulation of the movement of water and a dissolved solute from a point source into the soil profile, which contains only one layer inherently, prescribes itself to an axisymmetric situation. This is one of the reasons that Hydrus\_2D code was used since it has the capability of simulating axisymmetric flows. The following boundaries were considered for this analysis. Because of symmetry, a vertical cross section (ABCDE, Fig. 2), along the radius of the soil sample, was selected for simulation by the computer model. The boundary conditions for the cross section were shown in Figure 2 are as follows:

AB= constant flux boundary, represents the saturated water entry radius. BC, CD, DE and EA = no flow boundary. The boundary EA was a no flow boundary due to symmetry

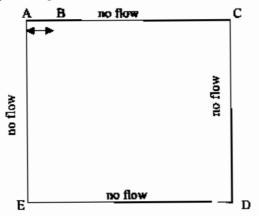


Figure 2: Schematic diagram showing the physical layout of the trickle irrigation system implemented in Hydrus\_2D, including the boundary conditions. The upper solid fine indicates the saturated water entry radius.

The boundaries BC, CD and DE were selected as no flow because in the actual experiment no water reached these boundaries. No flow boundary conditions is a special case of Neumann type boundary conditions where Darcy's law relates flow to gradient of head where.

$$\frac{\partial h}{\partial r} = \frac{\partial \theta}{\partial r} = 0 \qquad \frac{\partial h}{\partial z} = \frac{\partial \theta}{\partial z} = 0 \tag{6}$$

A constant boundary condition was selected for the leftmost corner nodes of the domain to represent the point of application of water.

#### **Initial and Boundary Conditions**

In order to investigate the influence of trickle discharge, soil hydraulic properties and a constant flux boundary condition at the saturated water entry radius were used.

The initial condition that must be satisfied within the flow domain to be uniform throughout the soil profile

$$\theta(\mathbf{r},\mathbf{z}) \ t = 0 \tag{7}$$

and the boundary conditions

$$\partial h/\partial r$$
 for  $z\ge0$ ,  $r=0$  and  $\partial h/\partial r=0$  for  $z\ge0$ ,  $r=R$  (8)

During water application, the point source boundary will have a constant saturated water entry radius. The water application rate becomes:

$$q=Q/A$$
  $(A=m^2s)$  (10)

where Q= the flow rate from dripper  $[L^3/T]$ , A = the area of surface saturated water entry  $[L^2]$   $r_3$  = saturated water entry radius [L]

#### Statistical Criteria

To evaluate the model's performance several measures were used, such as mean bias error, MBE, root mean square error RMSE and Theil's Inequality Coefficient U. The relationships describe these measures of analysis are

$$MBE = \frac{1}{n} \sum_{i=1}^{n} (P_i - A_i)$$
 (11)

RMSE = 
$$\sqrt{\frac{1}{n}\sum_{i=1}^{n}(P_{i}-A_{i})^{2}}$$
 (12)

$$U = \frac{\sqrt{\frac{1}{N} \sum_{i=1}^{N} (P_i - A_i)^2}}{\sqrt{\frac{1}{N} \sum_{i=1}^{N} (P_i)^2 + \sqrt{\frac{1}{N} \sum_{i=1}^{N} (A_i)^2}}}$$
(13)

Where  $P_i$  = the predicted data from the model,  $A_i$  = the experimental data and n = the number of records, (Naylor 1970 and Hossein et al., 2004). The MBE, RMSE statistics have as lower limit the value of zero, which is the optimum value for them as it is for U Theil's.

## RESULTS AND DISCUSSION Wetting Front Movement

Figures 3 and 4 show the actual and the computed surface wetted radius as a function of elapsed time in minutes for sandy soil under 1.5, 2, 2.7, 3, 3.5, 4.5, 4.8 and 6 l/h application rates. In these figures the open circles represent the actual location of the wetted front, while the solid circles represent the predicted. The figures indicate good simulation capability of the model in the horizontal direction, especially in the earlier stage of irrigation. The discrepancies between the numerical and the experimental surface wetted radius are generally small and may be attributed to the lack of precision in estimating soil hydraulic characteristics and measurement errors. Haverkamp et al., (1977) reported that a relative difference of only 2% in surface water content caused a relative difference of 24% in the wetting front position. Additional measures were used to evaluate the model performance, such as: mean bias error, MBE, root mean square error, RMSE and Their's coefficient, U. The MBE RMSE statistics have as a lower limit the value of zero, which is the optimum value for them, as it is for Theil's coefficient, U. These statistical parameters are summarized in Table 2. The MBE (ranged between 1.21and 2.83 cm) and RMSE (ranged between 1.49cm and 3.03 cm) are small, which indicates slight deviations between the numerical and experimental results. The values of U Theil's (ranged between 0.035 and 0.075) put more evidence the fact that the predicted values are close to zero which, in turn, reflect that the model has a good performance to simulate wetting front movement under point source trickle irrigation under the mentioned boundary conditions.

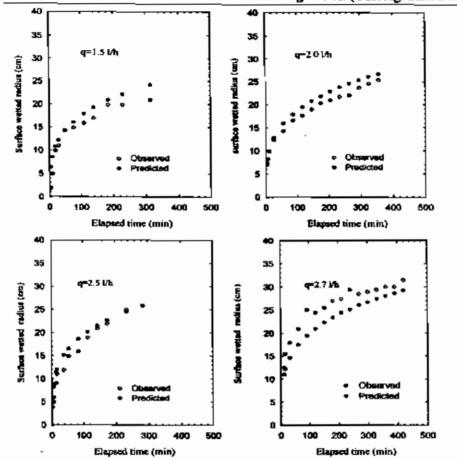


Figure 3: Observed and computed horizontal surface wetted radius for sandy soil under application rates of 1.5, 2.0, 2.5 and 2.7 l/h

Table 2: Values of the statistical parameters used in comparison for surface wetted radius for sandy soil

Discharge rate (Vh)	MBE (cm)	RMSE (cm)	Ų
1.5	1.62	2.29	0.075
2.0	1.21	1.49	0.040
2.5	1.51	1.95	0.059
2.7	-2.59	2.95	0.061
3.5	-2.83	3.03	0.061
4.5	-1.87	2.26	0.046
4.8	-1.23	1.63	0.035
6.0	-1.54	2.10	0.049

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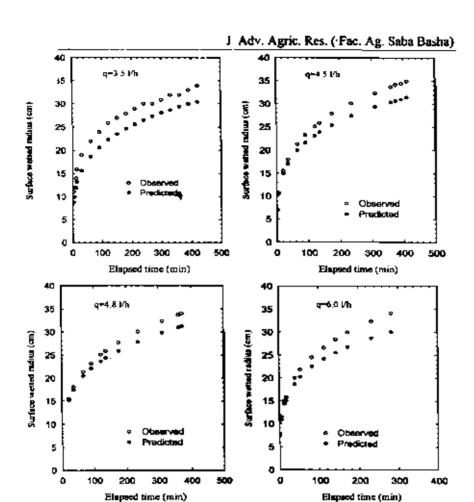
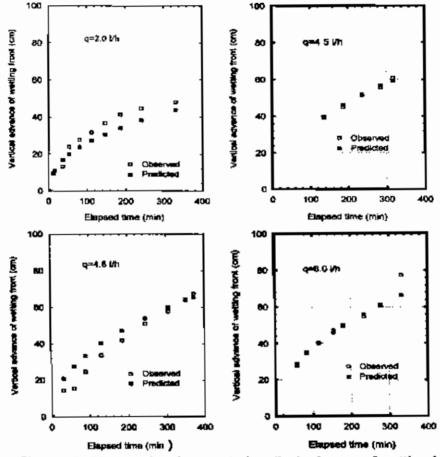


Figure 4: Observed and computed horizontal surface wetted radius for sandy soil under application rates of 3.5, 4.5, 4.8 and 6.0 i/h

The measured and simulated vertical advance of wetting fronts throughout the sandy soil profile under application rates of 2.0, 4.5, 4.8 and 6.0 Wh are shown in Figure 5. These values were measured along the vertical distance under the point source (r=0).

The simulated wetted vertical distances were defined as the depth of the wetted soil where maximum gradient in water content  $(\nabla \theta_i)$  occurred. As can be seen from these figures excellent overall agreements were achieved. The simulated vertical advances of wetting front were nearly identical to the observed values. For example, under 4.5 l/h application

rate, the simulated and measured values at 280 min were 56.0 cm and 56.6 cm, respectively. Similarly, under application rate of 6.0 l/h, the discrepancy between simulated and predicted value was 0.3 cm at 280 min.



Figures 5: Observed and computed vertical advance of wetting front for sandy soil under application rates of 4.8 and 6.0 l/h

The discrepancies can be attributed to the impact of surface topography where there was no guarantee that point source infiltration area would be exactly circular. On the other hand the natural variability of the soil properties and the lack of determination soil properties which determined under laboratory conditions may cause such these discrepancies. The adequacy indicators of the model were calculated. Table 3 displays the values of the mean bias error MBE, the root mean

square error RMSE and U Theil's coefficient for each treatment. The values of the mean bias error fluctuate from -2.95 cm to 4.21 cm, indicating an over all agreement between predicted and observed data. Also the values of RMSE and U Theil's can be considered small as their values are close to zero. This support the results mentioned above that Hydrus\_2D has the power to simulate the wetting front movement for sandy soil under point source trickle imigation.

Table 3: Values of the statistical parameters used in comparison for vertical advance of wetting front for sandy soil

Discharge rate (i/h)	MBE (cm)	RMSE (cm)	U' Thiet's
2.0	-2.95	4.39	0.076
4.5	0.16	0.95	0.010
4.8	4.21	5.92	0.066
6.0	-1.41	3.81	0.039

#### **Moisture Content Distribution**

Figures 6 through 8 show the measured and simulated moisture content distribution for each trial. Each figure contains contour plots of the measured and simulated moisture content profiles. The contours in the measured profile were drawn using kriging interpolation algorithm. However the data are relatively sparse, one should not attach too much significance to contour details (Skaggs et al., 2004). Both measured and simulated results show that in highly permeable coarse-textured soils such as the sand used in this study, water drains easily and quickly because gravity dominates. The irregular shape of the contour lines in case of observed data shows the natural water pathways. The other factor which may be the main reason for such discrepancies is the input data, i.e., the values of K., a, n and i, determined in laboratory, which normally differ with field data. Nevertheless, it is clear from the contour plots, the predicted pattern of wetting are in good agreement with the spatial distribution of the moisture content. The mean bias error MBE, the root mean square error RMSE and U Theil's coefficient for the simulated and measured volumetric water contents provide a quantitative measure of the goodness of fit between the observed and simulated data. Table 4 shows the values of model performance indicators for each trial. As can be noted from the Table, the values of model performance indicators are relatively approaching to their optimum. For instance, the MSE values range from 0.0006 to 0.040 m3 m3 while the values of RMSE oscillates between 3 313 and 0.06.In addition.

the Theil's coefficient values were found to be between 0.120 and 0.054. Although there are some relatively high values of these indicators, it can be acceptable according to the complex mechanisms of water transport under the complicated boundary and initial conditions from a surface point source

Table 4.: Values of the statistical parameters used in comparison for moisture distribution for sandy soil

Discharge rate	MBE	RMSE	U' Thiel's
( I/h)	$(m^3 m^{-3})$	(m³ m-³)	
1.5	0.0026	0.017	0.055
2.0	0.0057	0.037	0.086
2.5	0.0025	0.021	0.054
2.7	0.0097	0.029	0.073
3.5	0.0210	0.048	0.103
4.5	0.0429	0.060	0.128
4.8	0.0142	0.035	0.071
6.0	0.0006	0.062	0.120

#### CONCLUSION

The numerical HYDRUS\_2D model was used to simulate water movement in sandy soil under point surface trickle irrigation. The simulation of water movement was conducted for surface wetted radius, vertical advance of wetting front and the distribution of moisture content in the soil profile. Simulation positions of the wetting front were in agreement relative to the observed measurements of the wetting front. Specifically, in the lateral, the experimentally determined wetting front was closely estimated by the model. However, in the downward direction the simulated wetting front advanced much slower than the observed especially at later stage of infiltration. Considering the difficulties in estimating the dynamic water conditions in the field there was generally good agreement (especially in the lateral direction) between the measured and simulated values.

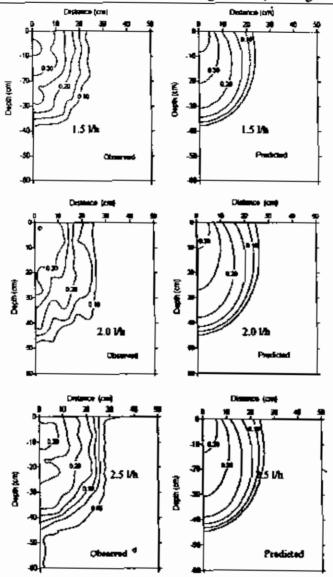


Figure6: Observed and predicted moisture distribution under application rate of 1.5 l/h, 2.0 and 2.5 elapsed time 320,360 and 429 minutes, respectively.

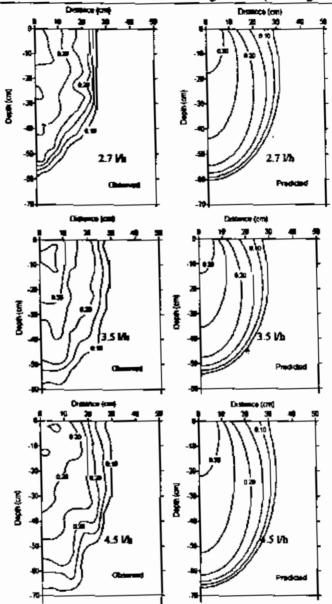


Figure 7: Observed and predicted moisture distribution under application rate of 2.7, 3.5 and 4.5 elapsed time 420, 420 and 370 minutes, respectively.

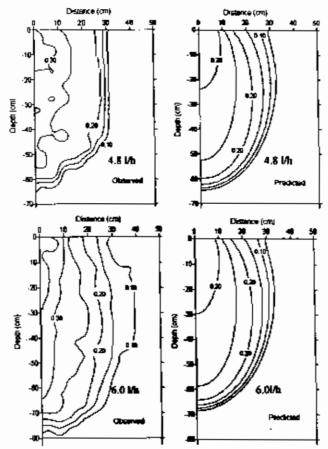


Figure 8: Observed and predicted moisture distribution under application rate of 4.8 and 6.0 l/h, elapsed time 370 and 340 minutes, respectively.

In the deeper downward direction the simulated moisture content distributions were less than the measured. On the other hand, the Hydrus\_2D model described the water content distribution quite well at relatively high levels of moisture contents; however, it did not do as well at lower moisture content. However, the results provide support for using HYDRUS-2D as a tool for investigating and designing drip irrigation management practices.

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#### الملخص العربي

### محكاة حركة الماء في الإراضي الرملية تحت ظروف حقل البراري باستخدام نموذج هيدرس ذات البعدين

احمد ابراهیم لخماج و عبدالرزاق مصباح عبدالعزیز قسم النربة والمداه – کابة الزراعة – جامعة الفاتح – طرابلس – نیبیا

وتتحرك جتنبيا ورامساء وقد وجدت فرضيات لشكل وتوزيع للرطوبة داخل حجسم للمساء العبتسل هميذه المعرفة تعتبر هامة في تصميم وتشغيل وادارة نظم الري بالتنقيط. وقد اجريت دراسة محاكاة توزيع مساء الغزية الرملية بمعدلات ٢٠٠، ٢٠٠، ٢٠٠، ٣٠٠، ٣٠٠، ٢٠٥، ٢٠٥ و ٢٠٠ لتز /ساعة. وقد كم قيالس قطر السطح المبتل: التقام الراسي لمجهة الابتلال، وتوزيع للمحتوى الرطوبي في قطـــاع التربـــة. وقـــــ المتخدمات ثلاث صفات لحصائية المقارنة جودة نتائج المحاكاه، وهي مترسط خطساً التحييز (MBE). الجذر التربيعي لمتوسط خطأ (RMSE) ، و معامل Theil لمعدم النساوي (U). وقد كانت لماكن جبهـــة الابتلال النائجة من عملية المحلكاة متوافقة نصيبا مع النتائج المقامة، خصوصا في الحركة الجانبية التي تم تقدير ها بواسطة النموذج. لهما بالنسبة للحركة الى لعظ، فقد كان معدل تقدم الجبهة اكثر بطأ من النشافج المقاسة، وخصوصًا في العراجل النهائية من عملية الرشح، هذا مع الاخذ في الاعتبار صحوبات تقديرًا ديناميكية غروف الماء في الحقل. وبالنسبة لمحركة الماء العميقة الى اسفل، فقد وجد ان توزيع المحتــوى الرطوبي المصوب من عملية المحاكاة قل من ذلك المقاس. على الجانب الاخر، فقد وصدَّف نموذج هيدرس ذات البعدين توزيع المحتوى الرطوبي بدرجة جيدة ، على السرغم مسن غيساب ذلسك بالنسبة للمحتويات الرطوبة المنخفضة. ويعزو الاختلاف بين القيم المقاسة والقيم التمحصل عليهما من عمليمة المحاكاة في اختلاف مصدر فمواه فتناء الرشح وكانك الاختلاف في خوامس فتربة فطبيعية. وتزيد النثلج فستخدام نموذج هيدرس ذات اليعدين (Hydrus 2D) كاداة لقمص وتصميم شبكات الراق بالتتقيط، علمي الرغم من المبكلتيكية المحدة لحركة الماء تحت طروف حالة الحدود والحالة الإولية.