Evaluation of Some Crop Yield Response Functions

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ABSTRACT

This study provides an evaluation of a number of crop yield response functions. The relative sensitivity of each function in application with the same water stress has been assessed. The coefficients for each function were derived from the linear relationships between relative yield and relative evapotranspiration as described by Doorenbos and Kassam (1979), The data showed that the stage-by-stage results are approximately the same using all functions. When using published crop yield coefficients, all functions gave approximately the same results.

Keywords: ***Crop yield, Water deficit, Soil water stress***

INTRODUCTION

Crop water production functions describe crop yield response to water availability. Many crop water production functions have been developed (de Wit, 1958; Jensen, 1968; Minhas *et al.*, 1974; Stewart *et al.*, 1976; Doorenbos and Kassam, 1979; Martin *et al.*, 1984; and Rao *et al.*, 1988). Many crop water production functions have the same basic form, in which crop yield is linearly related to cumulative actual evapotranspiration for the crop. Table 1 presents some well-known crop water production functions. The existing functions are of two types: additive (Stewart *et al.*, 1976) and multiplicative (Jensen, 1968; Minhas *et al.*, 1974; Rao *et al.*, 1988). The Stewart and Rao functions are quite similar and divide the growing season into a number of stages with different yield response factors (K_y). These two functions use the yield response factors published in the FAO Irrigation and Drainage Paper No. 33 (Doorenbos and Kassam, 1979), which depend upon the crop type and its stage of growth. The main difference between these functions is that the Stewart function is additive whereas the Rao function is multiplicative. The Jensen and Minhas functions use a crop sensitivity factor (λ_i) for each stage of growth. λ is used as an indicator for the effect of water stress on crop yield. The higher the value of λ the greater the effect of water stress. The Doorenbos and Kassam, and Martin functions have exactly the same form in which K_y and b_e represent the slope of the linear relationship between relative evapotranspiration and relative yield. These functions cannot be used to evaluate the cumulative effect of water stress during the different growth stages. They can only be used to quantify the crop response to water at the end of a particular stage, or for the whole growing season. Crop water production functions can be used for evaluating alternative irrigation scheduling strategies especially when water is a limited resource. The effect of water stress on crop yield and growth is dependent on the stage of growth during which the stress occurs. As a result, many irrigation scheduling strategies apply adequate amounts of water during the sensitive periods of crop growth to avoid high yield reduction. In these strategies, water applications can be reduced during the less sensitive stages with lower yield reduction. Although yield response functions have been extensively used to quantify the yield-water relationship, there are still issues, particularly in relation to stages of growth. There is still a gap in knowledge in terms of the cumulative impact of water stress on crop yield in the different growth periods. Also, some of these functions are valid only for the conditions under which they were developed and parameters of the functions are location specific. As a result, some of these functions cannot be applied for irrigation water management in locations with different conditions from those where they were developed. This study provides an evaluation of a number of crop water production functions to assess their relative sensitivity in application with the same water stress. The accuracy of the functions in calculating crop yield under different levels of evapotranspiration deficit is also examined.

BACKGROUND

The relationship between crop production and evapotranspiration is called the "crop water production function" and in effect it defines crop yield response to different levels of water applied. A crop water production function can be used to evaluate water use efficiency and to assist in effective allocation of water resources. Many crop water production functions have been developed and used to quantify crop yield response to water.

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Table 1. Some well-known crop water production functions

Water production function	Author	Type
$\frac{Y_a}{Y_m}=\prod_{i=1}^{n}\left(\frac{ET_a}{ET_m}\right)^{\lambda i}$	Jensen (1968)	Multiplicative
$\frac{Y_a}{Y_m}=\prod_{i=1}^{n}\left[1-\left(1-\frac{ET_a}{ET_m}\right)_i^2\right]^{b_i}$	Minhas et al., (1974)	Multiplicative
$\frac{Y_a}{Y_m}=1-\sum_{i=1}^{n}K_i\left(1-\frac{ET_a}{ET_m}\right)_i$	Stewart et al., (1976)	Additive
$\left(1-\frac{Y_a}{Y_m}\right)=K_y\left(1-\frac{ET_a}{ET_m}\right)$	Doorenbos and Kassam (1979)	-
$\frac{Y_a}{Y_m}=(1-b_e)+b_e\left(\frac{ET_s}{ET_m}\right)$	Martin et al., (1984)	-
$\frac{Y_a}{Y_m}=\prod_{i=1}^{n}\left[1-K_i\left(1-\frac{ET_a}{ET_m}\right)_i\right]$	Rao et al., (1988)	Multiplicative

Jensen (1968) proposed a multiplicative function to determine the cumulative effect of water shortage in different stages of crop growth. The function used is of the form:

$$\frac{Y_a}{Y_m}=\prod_{i=1}^{n}\left(\frac{ET_a}{ET_m}\right)^{\lambda i} \quad (1)$$

where, Y_a is the actual crop yield, Y_m is the maximum or potential crop yield, ET_a is the actual crop transpiration, ET_m is the potential crop transpiration, i is stage of growth, and λ_i is the crop sensitivity factor for growth stage i. λ_i is an indicator for the effect of water stress on crop yield. The greater the value of λ_i the greater the effect of water shortage. A limitation of the Jensen function is in determining appropriate values of λ_i.

λ_i can be determined graphically (Tsakiris, 1982). For any particular growth stage:

$$\frac{Y_a}{Y_m}=\left(\frac{ET_{ai}}{ET_{mi}}\right)^{\lambda_i} \quad (2)$$

and in logarithmic space:

$$\log\frac{Y_a}{Y_m}=\lambda_i\log\left(\frac{ET_{ai}}{ET_{mi}}\right) \quad (3)$$

The problem with this method is in determining relative yield in different growing stages. Actual yield is not known until harvest. Tsakiris (1982) used relative yield values at different growth stages under different evapotranspiration deficits presented in the FAO Irrigation and Drainage Paper No. 33 (Doorenbos and Kassam, 1979) to permit calculation of values of λ_i, that was conditioned entirely on the Doorenbos and Kassam (1979) data, the origins of which are unclear. Doorenbos and Kassam (1979) suggested that the following function be used to determine the crop yield response to water through various stages of crop growth:

$$\left(1-\frac{Y_a}{Y_m}\right)=K_y\left(1-\frac{ET_a}{ET_m}\right) \quad (4)$$

where, K_y is the yield response factor depending upon the crop and the stage of crop growth. However, no guidance is given by Doorenbos and Kassam on how to apply the yield response function when water shortage occurs cumulatively during different stages of crop growth (Wardlaw and Barnes, 1996; Rao *et al.*, 1988). Values for K_y are published by Doorenbos and Kassam for many crop types to determine the yield reduction over the whole growing season and for individual growth stages. Rao *et al.*, (1988) used the same procedure as Tsakiris to calculated λ_i for each value of K_y listed by Doorenbos and Kassam (1979). Values of λ_i corresponding to different values of K_y are presented in Table 2.

Table 2.Values of crop sensitivity factor (λ_i) corresponding to different values of crop type (K_y)

K_y	0.20	0.25	0.30	0.40	0.45	0.50	0.55	0.60	0.75	0.80	1.00	1.50
λ_i	0.15	0.19	0.24	0.32	0.37	0.42	0.47	0.52	0.68	0.74	1.00	1.95

Source: Rao *et al.*, (1988)

Zhang and Oweis (1999) used the Jensen function to investigate the effect of water deficit during certain growth stages on wheat yield in Tel Hadya in northern Syria. Non-linear regression procedures were used to find λ_i. They found that crop yield increased linearly by 160 $kg\ ha^{-1}$ for bread wheat and 116 $kg\ ha^{-1}$ for durum wheat per 10 mm increase in evapotranspiration above a threshold of 200 mm. The sensitivity index λ_i values for wheat at the different growing stages are presented in Table 3.

Minhas *et al.*, (1974) proposed the following multiplicative type function:

$$\frac{Y_a}{Y_m}=\prod_{i=1}^{n}\left[1-\left(1-\frac{ET_a}{ET_m}\right)_i^2\right]^{b_i} \qquad (5)$$

where, b_i is the sensitivity factor for growth stage i. b_i can also be derived using relative yield values at different growth stages under different evapotranspiration deficits presented in Doorenbos and Kassam (1979).

Stewart *et al.*, (1976) proposed the following additive function:

$$\frac{Y_a}{Y_m}=1-\sum_{i=1}^{n}K_y\left(1-\frac{ET_a}{ET_m}\right)_i \qquad (6)$$

This function uses the same K_y as published in the FAO Irrigation and drainage Paper No. 33 by Doorenbos and Kassam, (1979). Wardlaw and Barnes (1996) questioned the applicability of the function, demonstrating that the Stewart function indicated almost total yield loss with 20% and 15% reduction in the actual evapotranspiration of rice and dry foot crops respectively.

Martin *et al.*, (1984) suggested the following function for grain sorghum, wheat and soybean, which can be used to predict the relative yield of these crops:

$$\frac{Y_a}{Y_m}=(1-b_e)+b_e\left(\frac{ET_a}{ET_m}\right) \qquad (7)$$

where, b_e is an empirical seasonal yield coefficient.

Values for b_e are summarized by Martin *et al.*, (1984) for grain sorghum, soybean and hard red winter wheat only. Lamsal *et al.*, (1999) derived an experimental value of b_e for wheat:

$$Y_a/Y_m=1-1.07+1.07(ET_a/ET_m) \qquad (8)$$

A further multiplicative function was proposed by Rao *et al.*, (1988):

$$\frac{Y_a}{Y_m}=\prod_{i=1}^{n}\left[1-K_y\left(1-\frac{ET_a}{ET_m}\right)_i\right] \qquad (9)$$

The function is similar to that proposed by Minhas *et al.*, (1974), but is intended to be used with the crop yield response factors published by Doorenbos and Kassam. In assessment of crop yield response functions, Wardlaw and Barnes (1996) suggested that Rao function was at that time the best approach to crop yield response modelling. Results from the Jensen function (Jensen, 1968) were similar but parameterisation of the sensitivity factor is not as well defined as for the Rao function.

Rao *et al.*, (1988) compared the relative yield of cotton, groundnut, maize, sorghum, sunflower, and wheat predicted by different crop water production functions (Jensen 1968; Stewart *et al.*, 1976; and Rao *et al.*, 1988) against field data. They concluded that, although all three functions predicted identical values of relative yield at lower levels of soil moisture stress, the Stewart function gives very low relative yield values (sometimes negative) when using higher values of K_y even when the relative evapotranspiration deficit at any stage of growth does not exceed 50%.

Table 3. Sensitivity index (λ) of bread wheat and durum wheat to water stress at different growing stages at Tel Hadya in Syria

Crop	Seedling	Stem elongation-booting	Booting - anthesis	Anthesis-soft dough	Soft dough-maturity
Bread wheat	0.01	0.31	0.28	0.21	0.10
Durum wheat	0.15	0.31	0.17	0.26	0.07

Source: Zhang and Oweis, (1999)

Rao *et al.* (1988) also compared the crop yield predicted by different crop production functions with field data collected by several researchers (Hall and Butcher, 1968; Jensen, 1968; Hiler and Clark 1971; Hanks 1974; Minhas *et al.*, 1974; Blank, 1975; Stewart *et al.*, 1976; Sudar *et al.*, 1981). They reported that the parameters for many functions need to be determined locally. Their conclusion was that functions are valid under the conditions where they were developed.

RESULTS AND DISCUSSION

Assessment of crop yield response functions

An evaluation has been made of the crop water production listed in Table 1. In this evaluation, sorghum and cotton yields were calculated under different levels of water deficit. The Doorenbos and Kassam (1979) function was used in the calculation of the sensitivity factor (λ) for the Jensen function for the individual growth stages, following the procedure described by Tsakiris (1982).

The Stewart and Rao functions can be used with the crop yield response factors K_y published by Doorenbos and Kassam (1979). Since the Doorenbos and Kassam and Martin functions are exactly the same, the Martin function is considered no further. For the Jensen and Minhas functions the coefficients were derived from the Doorenbos and Kassam (1979) data also.

The Doorenbos and Kassam function has been used by irrigators, economists and policy makers to assess their plans and scenarios for investment in the agricultural sector. However, the FAO expert meeting on crop water productivity, held in February 2003 in Rome, considered it important that the function be revised to be able to quantify the crop yield response under very limited water supply. The parameters and procedures described in the FAO Irrigation and Drainage Paper No. 33 (Doorenbos and Kassam, 1979) require updating and refinements in order to reflect the increase in productivity of the major crops over the last 25 years (FAO, 2003).

The problem with using the Doorenbos and Kassam function is that it cannot be used to quantify the effect of water stress on crop yield cumulatively. No guidance is given by Doorenbos and Kassam on how to apply the yield response function when water shortage occurs cumulatively during different stages of crop growth. This function can only be used to predict reduction in crop yield at the end of each stage of growth separately as well as at the end of the growing season based on the crop reduction factors published in Doorenbos and Kassam, (1979). However, Smith (1992) in the CROPWAT program has used Doorenbos and Kassam function in a cumulative way where each yield reduction for a given stage is carried over to the next stage according to:

$$(1-Y_a/Y_m)_i = 1-(Y_a/Y_m)_1 * (Y_a/Y_m)_2 * \ldots (Y_a/Y_m)_i \quad (10)$$

where, 1,2…i is stage of growth.

Smith (1992) has basically used the same procedure as the Rao function. Adopting this procedure for the Doorenbos and Kassam function, an evaluation has been made of the functions listed in Table 1. With the exception of the Stewart function, all other functions produce exactly the same results. The Stewart function, which is the only additive function in this comparison, gives generally lower relative yields than other functions in the range of evapotranspiration deficits considered. Figures 1 and 2 show that the different functions behave in the same way on sorghum and cotton. The results presented are for the entire growing season, but are calculated on the basis of the same evapotranspiration deficit occurring in each growth stage. Figures 3 and 4 show the impact of using the Doorenbos and Kassam seasonal coefficients, rather than stage of growth coefficients. Higher relative yield is predicted than when the stage of growth coefficients are used. This is a shortcoming of the function, and the effect of water stress should be taken into account cumulatively over the different growth stages.

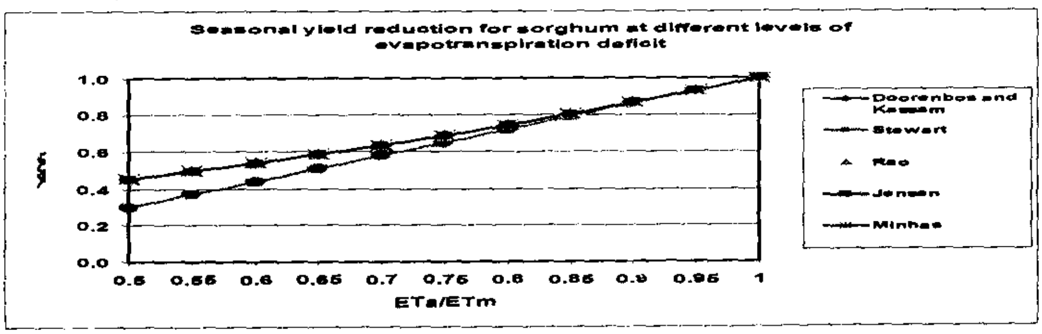


Figure 1. Comparison of yield response functions for sorghum

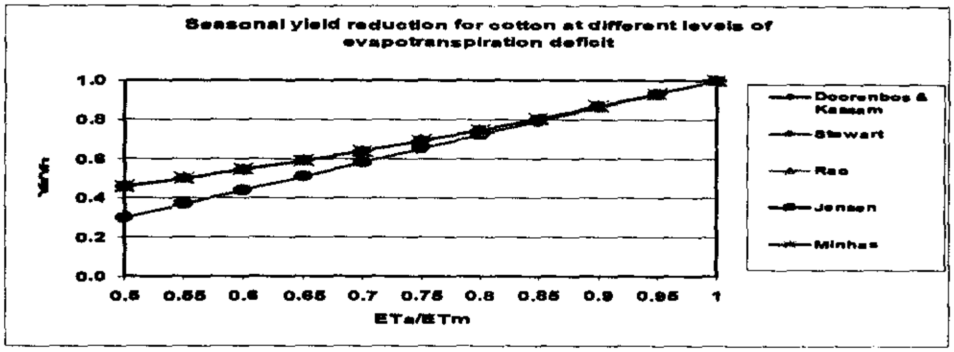


Figure 2. Comparison of yield response functions for cotton

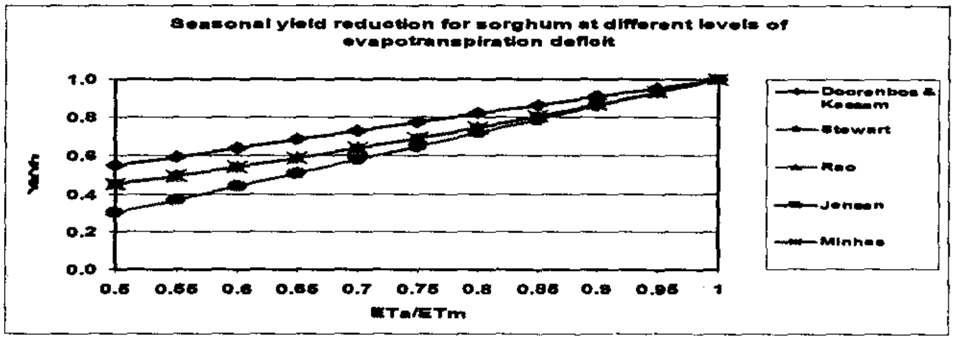


Figure 3. Comparison of yield response functions for sorghum using Doorenbos and Kassam seasonal coefficient

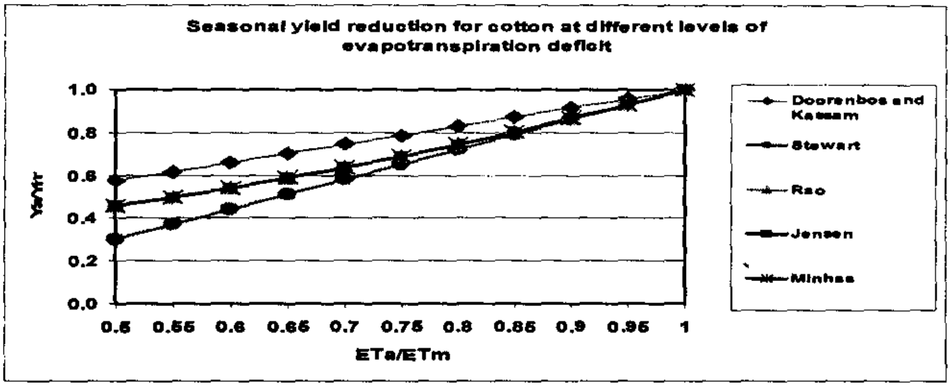


Figure 4. Comparison of yield response functions for cotton using Doorenbos and Kassam seasonal coefficient

Figures 5 and 7 show the relative yields for sorghum and cotton at each stage of crop growth with each of the functions, as well as the seasonal relative yield under 50% evapotranpiration deficit. 50% evapotranspiration deficit is the lower limit at which K_y was determined by the Doorenbos and Kassam. Since the coefficients for each function were derived from the linear relationships between relative yield and relative evapotranspiration published by Doorenbos and Kassam (1979), the stage by stage results are the same using all functions except Stewart function which gives a lower stage by stage relative yield as well as a lower seasonal relative yield than the Rao, Jensen and Minhas functions in flowering, yield formation and ripening growing stages. The Doorenbos and Kassam function gave exactly the same stage by stage relative yield as other functions as the effect of water stress on crop yield was applied cumulatively. However, when applied with seasonal coefficients, the Doorenbos and Kassam function gives a higher seasonal relative yield than the other functions. However, under 10% evapotranspiration deficit, all functions, gave approximately similar values of relative yield for the individual growth stages (Figures 6 and 8).

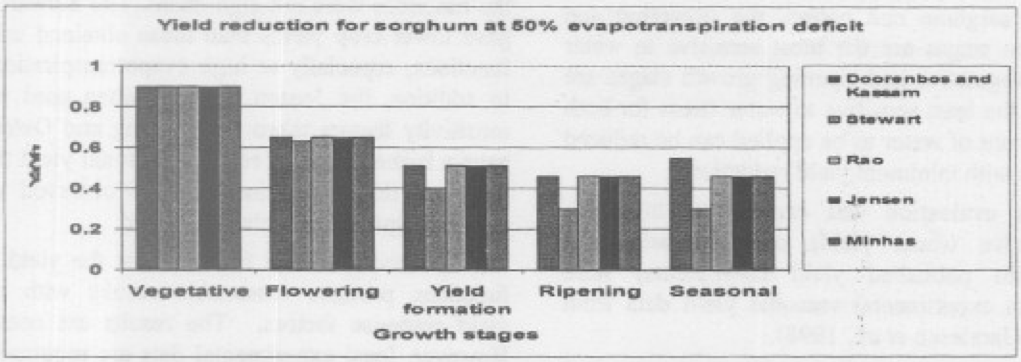


Figure 5. Sorghum yield reduction in different stages of crop growth

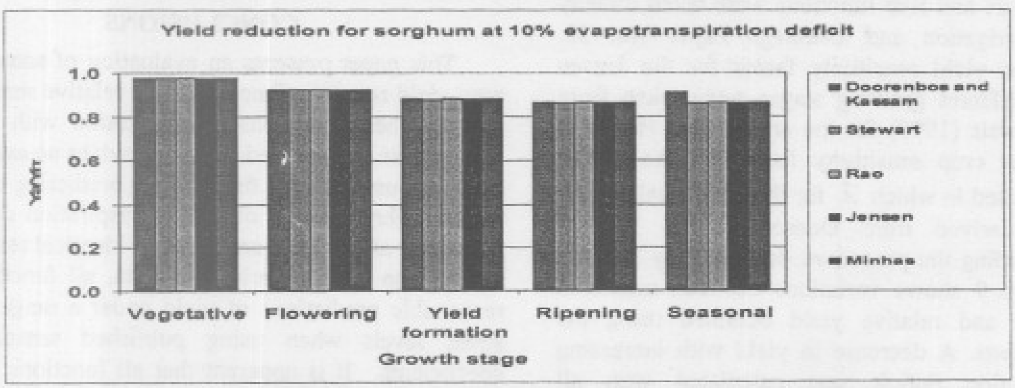


Figure 6. Sorghum yield reduction in different stages of crop growth

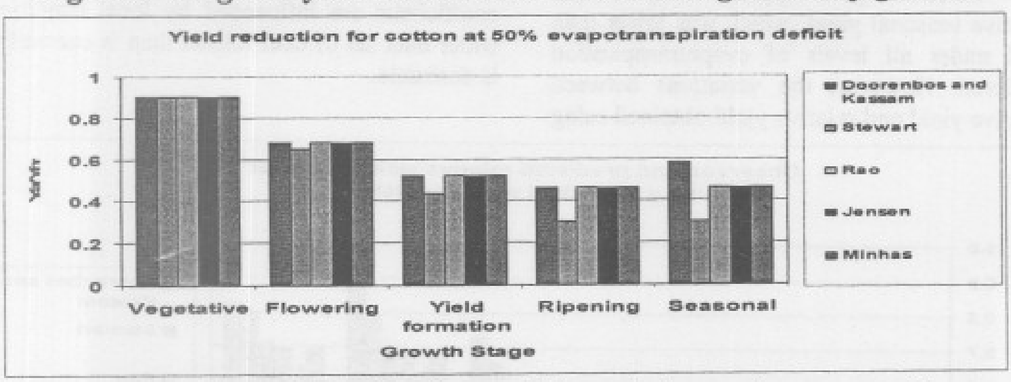


Figure 7. Cotton yield reduction at different stages of crop growth

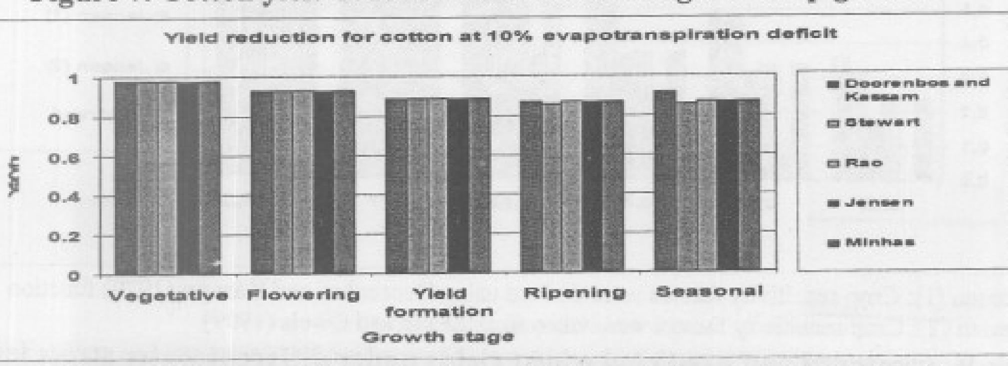


Figure 8. Cotton yield reduction at different stages of crop growth

For both sorghum and cotton, the flowering and yield formation stages are the most sensitive to water stress. Since vegetative and ripening growth stages are considered as the least sensitive to water stress for both crops, the amount of water to be applied can be reduced in these stages with minimum yield reduction.

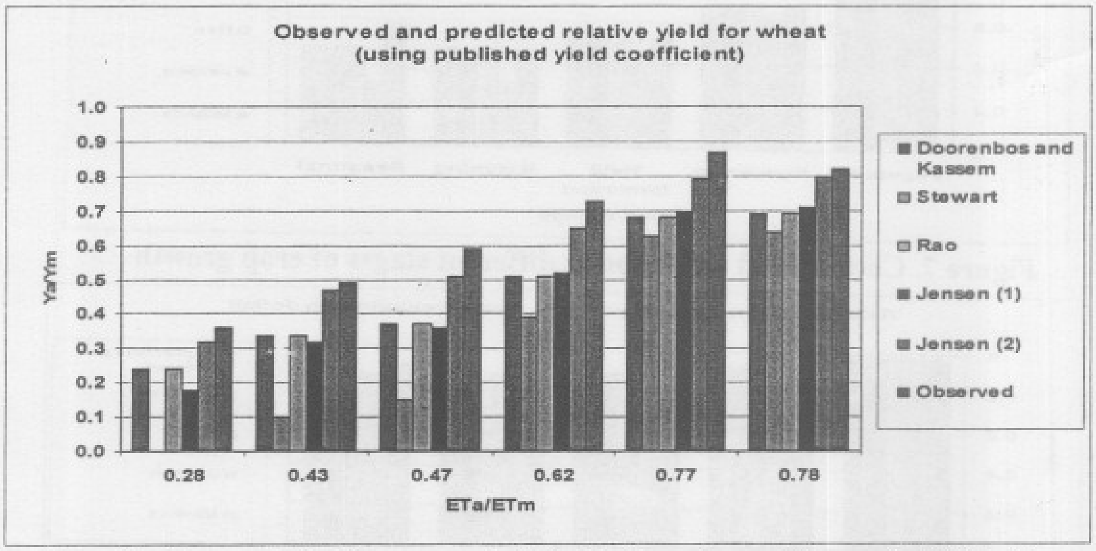
A further evaluation was carried out in which seasonal relative wheat yields obtained using the functions (with published yield coefficients) were compared with experimental seasonal yield data from New Zealand (Jamieson *et al.*, 1998).

The published yield coefficients for Doorenbos and Kassam, Stewart and Rao functions were taken directly from FAO Irrigation and Drainage Paper No. 33. However, crop yield sensitivity factor for the Jensen function at different growing stages were taken from Zhang and Oweis (1999) for the area of Tel Hadya in Syria. Another crop sensitivity factor for the Jensen function was used in which λ for the individual growth stages was derived from Doorenbos and Kassam function according the procedure described by Tsakiris (1982). Figure 9 shows variations between measured relative yield and relative yield obtained using the various functions. A decrease in yield with increasing evapotranspiration deficit was calculated with all functions. All functions gave approximately similar values of relative seasonal yield, which was lower than that observed under all levels of evapotranspiration deficit considered. However, the variations between measured relative yield and relative yield obtained using the functions were not significant. The Stewart function gave lower crop yields than those obtained using other functions, especially at high evapotranspiration deficit. In addition, the Jensen function when used with crop sensitivity factors taken from Zhang and Oweis (1999) gave a higher value of relative seasonal yield than other functions that were closer to the observed under all evapotranspiration levels considered.

The above results indicate that the yield response functions produce reasonable results with published yield response factors. The results are conservative. However, local experimental data are required in order to establish more reliable crop yield coefficients.

CONCLUSIONS

This paper presents an evaluation of some existing crop yield response functions. The relative sensitivity of each of these functions in application with the same water stress is presented. It also provides an examination of the accuracy of the functions in predicting crop yield under different levels of evapotranspiration deficit. In fact many of the functions produce identical results. In a comparison with experimental data, all functions gave reasonable predictions of yield under a range of water stress levels when using published seasonal yield coefficients. It is apparent that all functions would be suitable for yield response modelling. Yield response coefficients are influenced by local conditions, and a wider data set of coefficients than is currently available is desirable.



Jensen (1): Crop sensitivity factors were derived using Doorenbos and Kassam (1979) function
Jensen (2): Crop sensitivity factors were taken from Zhang and Oweis (1999)

Figure 9. Observed and predicted wheat yields under different water stress levels

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الملخص العربي

تقييم بعض الدوال المستخدمة في تقييم استجابة إنتاجية المحاصيل لكميات المياه المضافة

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يتضمن هذا البحث تقييم عدد من الدوال المستخدمة في دراسة استجابة إنتاجية المحاصيل لكميات المياه المضافة من حيث مقارنة الإنتاج الفعلي المحسوب باستخدام كل دالة من الدوال تحت مستويات متساوية من المياه المضافة لكل محصول من المحاصيل المستخدمة في هذا التقييم. ونظراً لأن معامل التأثير المستخدم في كل دالة من هذه الدوال مشتق من العلاقة الخطية بين الإنتاج النسبي والبخرنتح النسبي المنشورة في بحث Doorenbos and Kassam 1979، فأن الإنتاج المحسوب باستخدام هذه الدوال يكون تقريباً متساوي بإضافة مستويات مختلفة من المياه. ووجد أيضاً أن الأنتاج يكون تقريباً متساوي باستخدام معاملات تأثير أخرى غير تلك المنشورة في النشرة المذكورة أعلاه.